Lecture 14: Shape Google: Rigid Shape Statistics

COMPSCI/MATH 290-04

Chris Tralie, Duke University

3/1/2016

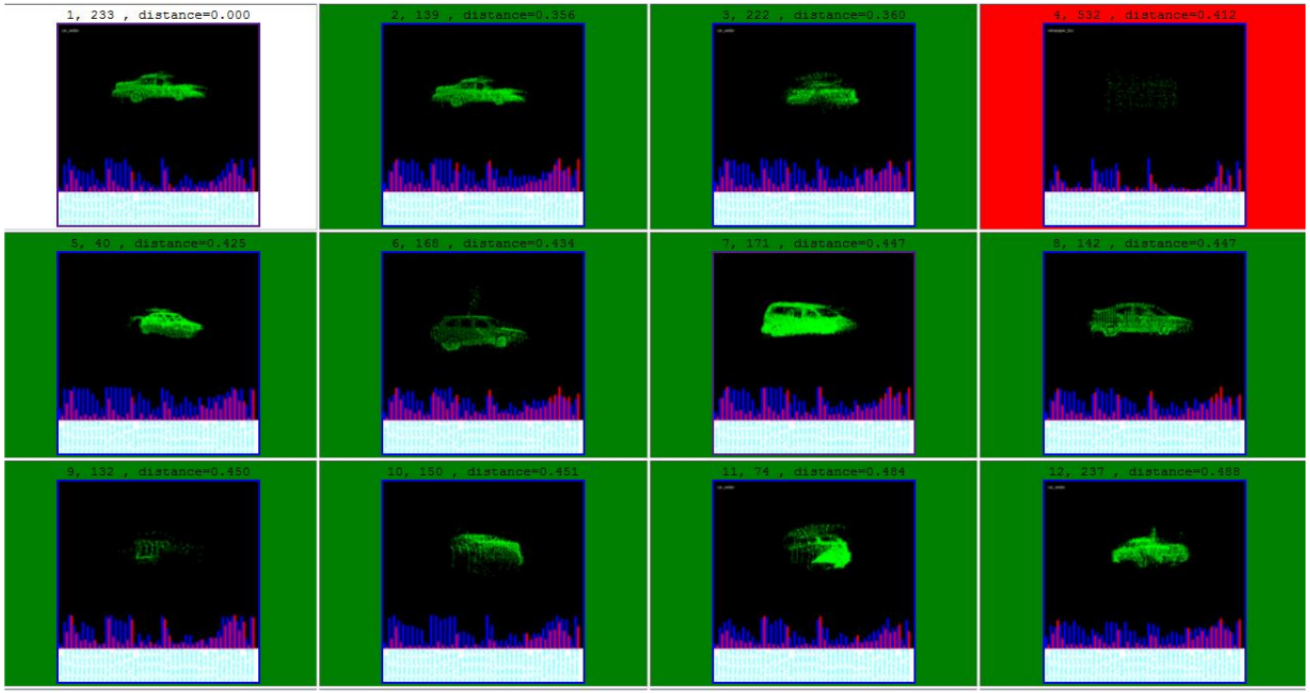
Announcements

- Group Assignment 1 Full Submission Due Tomorrow (Wednesday) 11:55 PM
- Rank Top 3 Final Project Choices By Friday 3/5 (Groups of 3-4)
- Attendance Policy Clarification
- Midterm Next Thursday 3/10

Table of Contents

Random Sampling By Area

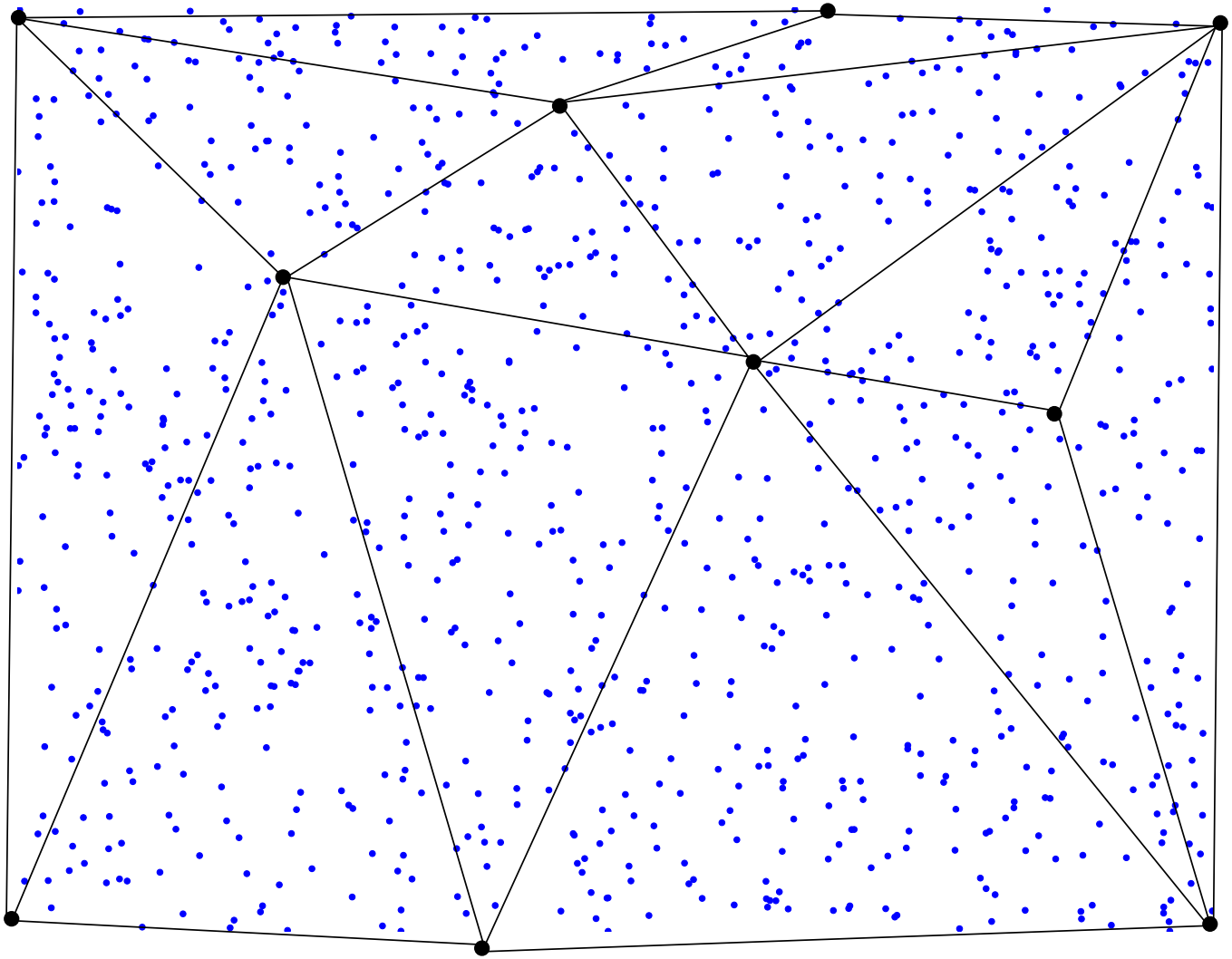
Goal: Given a shape, return similar shapes in a database



Tralie 2010

- Focus on point clouds
- Focus on shapes similar under *rigid motion*

Random Sampling By Area



Centroid Centering / RMS Scaling

For a point cloud $\{\vec{x}_i\}_{i=1}^N$

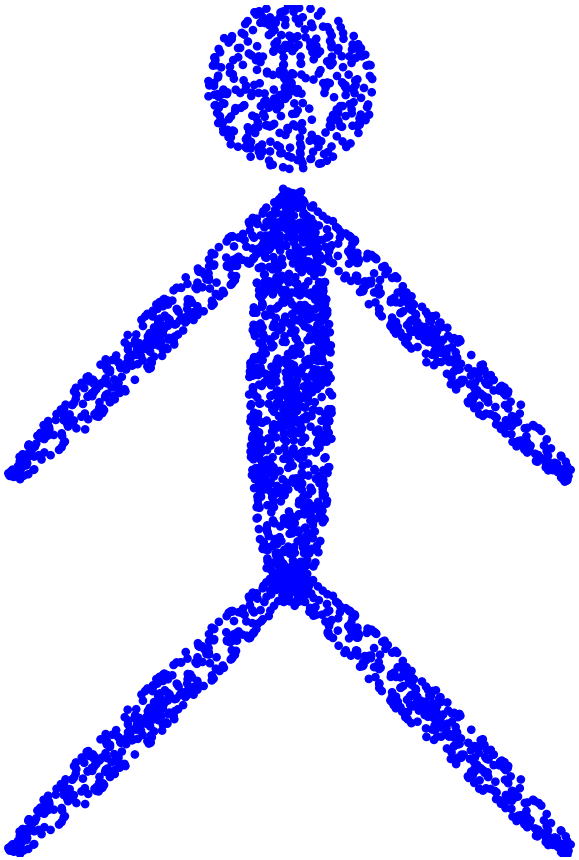
- ▷ Subtract off centroid
- ▷ Root mean square scale. Want

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}||\vec{x}_i||^2} = 1$$

Shape Matching Criteria

- Concise To Store
- Quick to compute
- Efficient to match
- Discerning
- Noise tolerant
- *Rotation Invariant*

Shape Histogram: Shells



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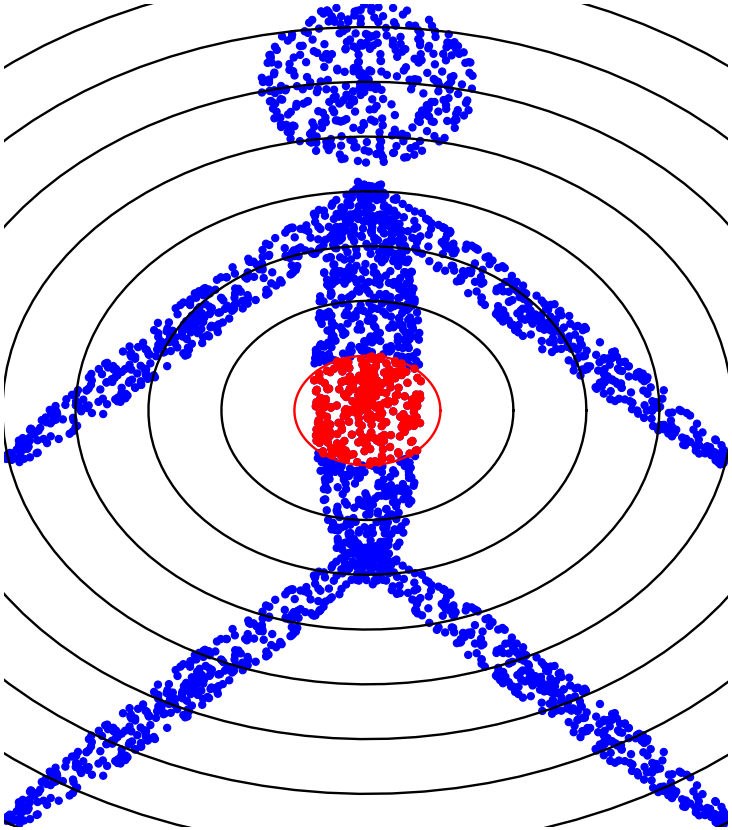

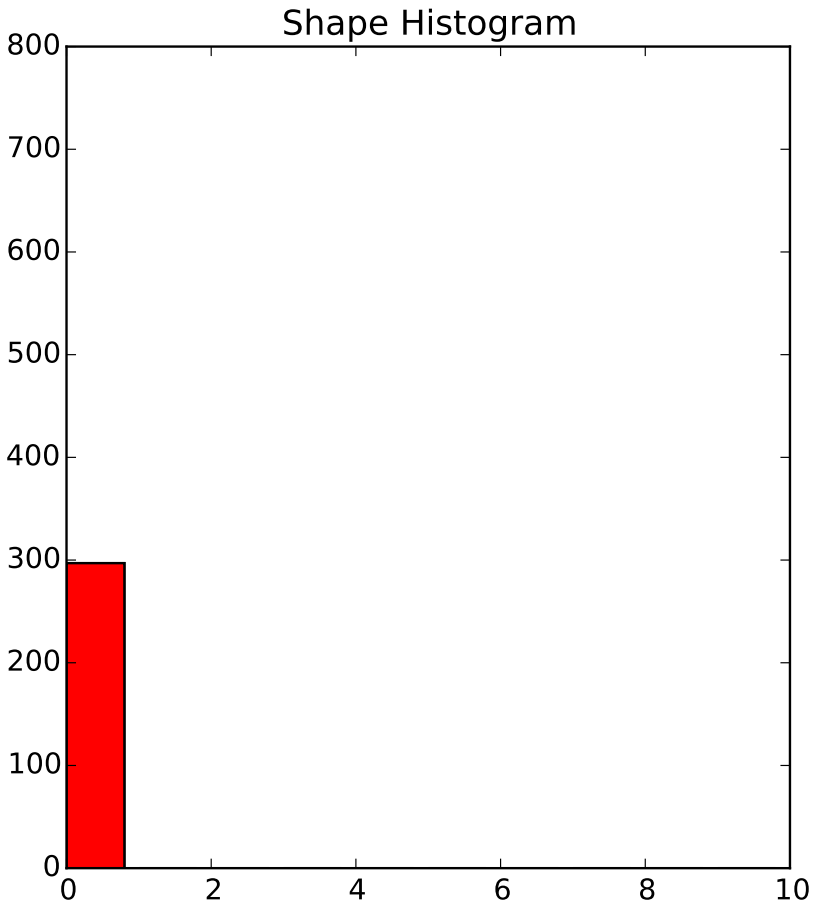

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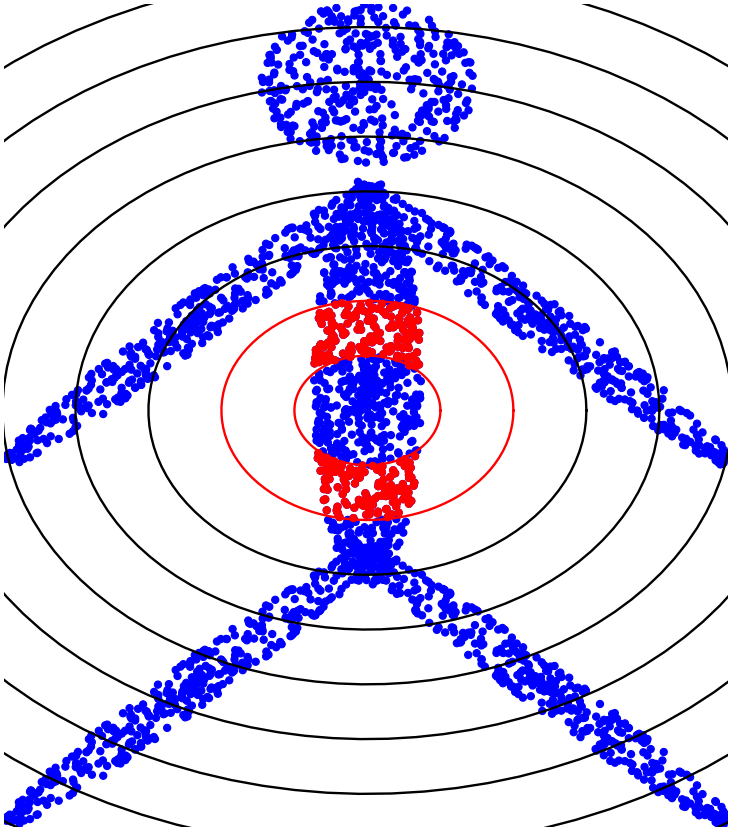

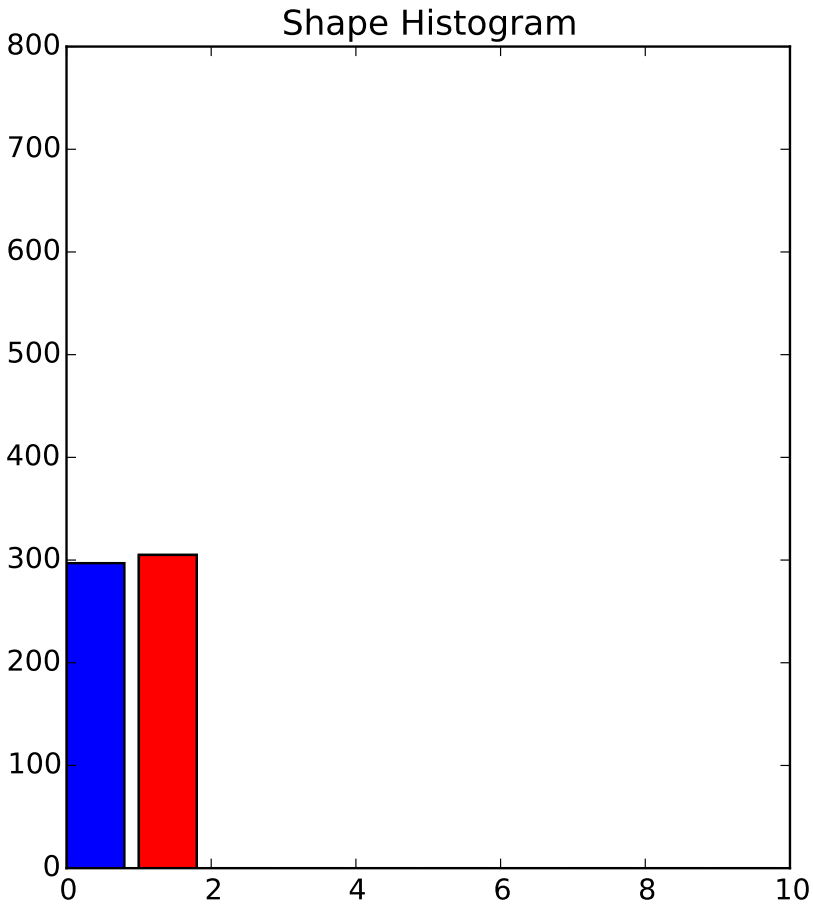

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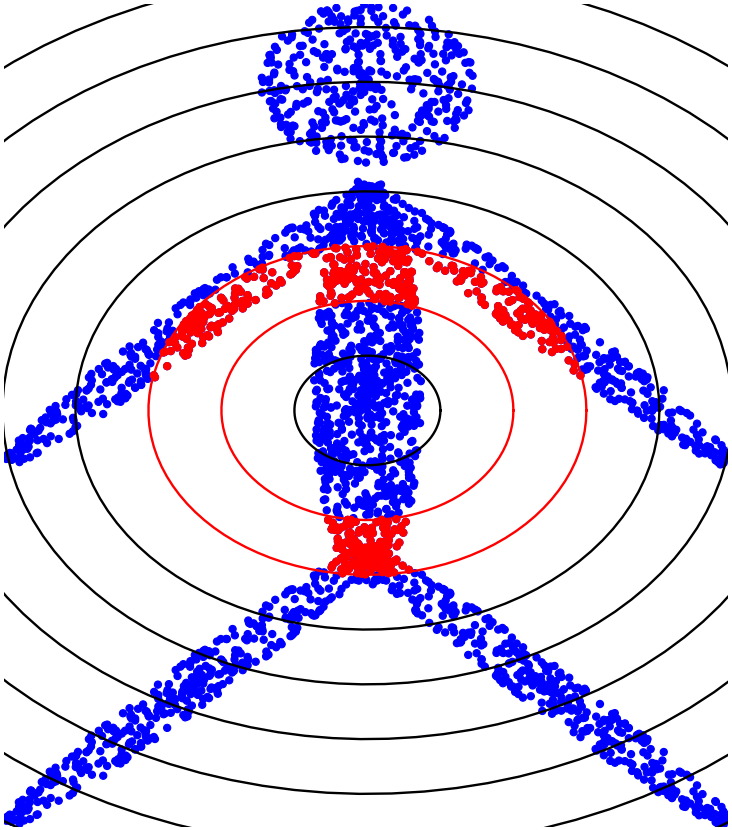

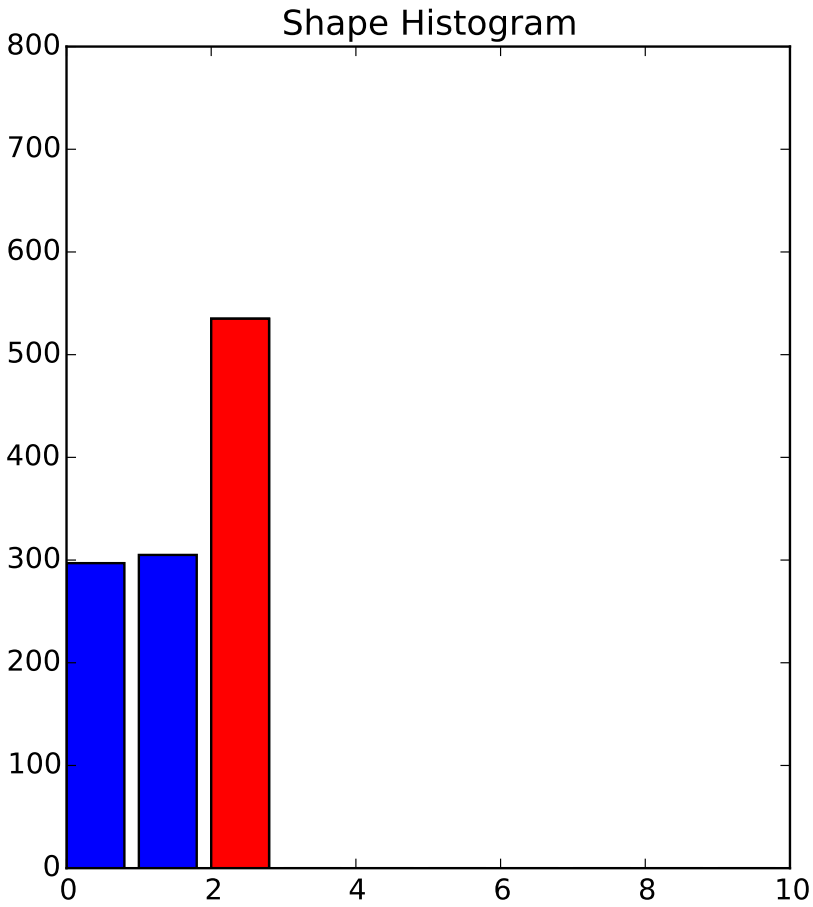

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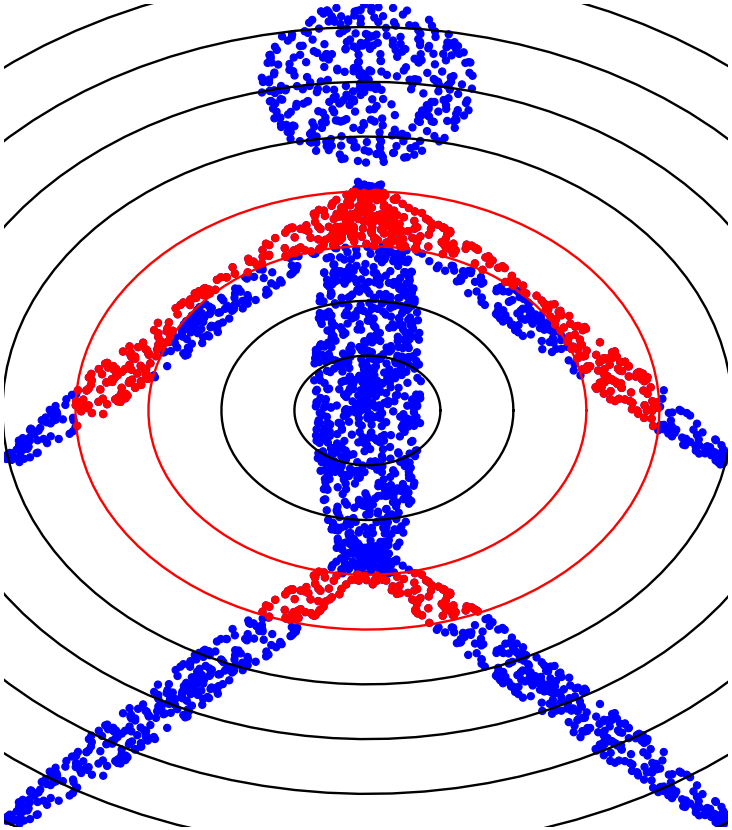

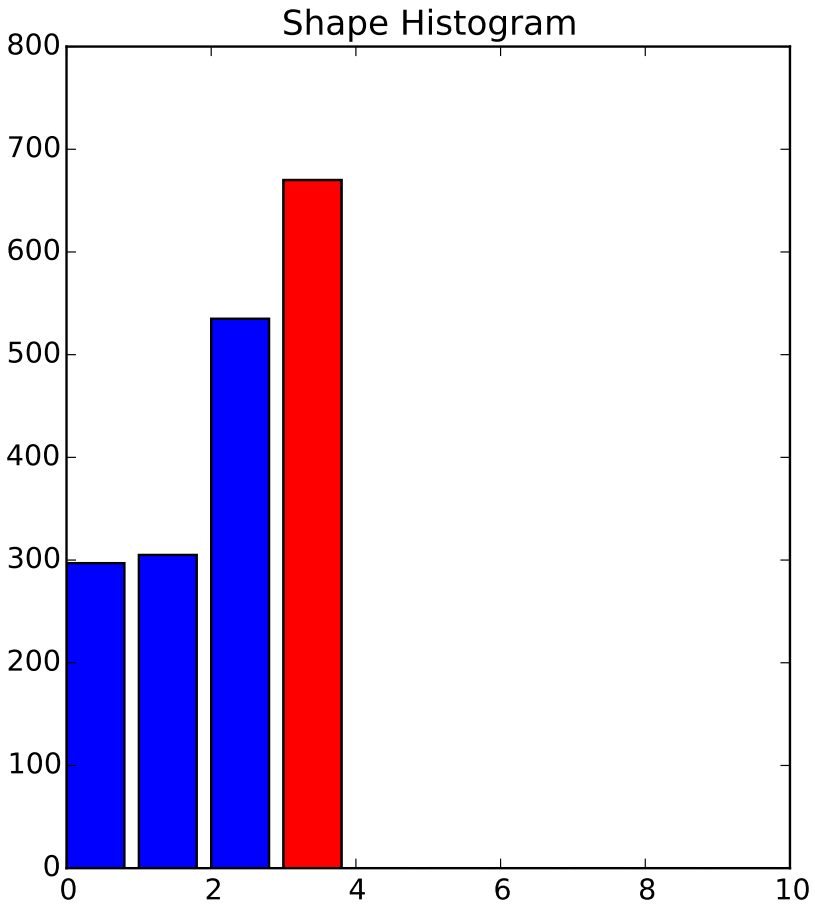

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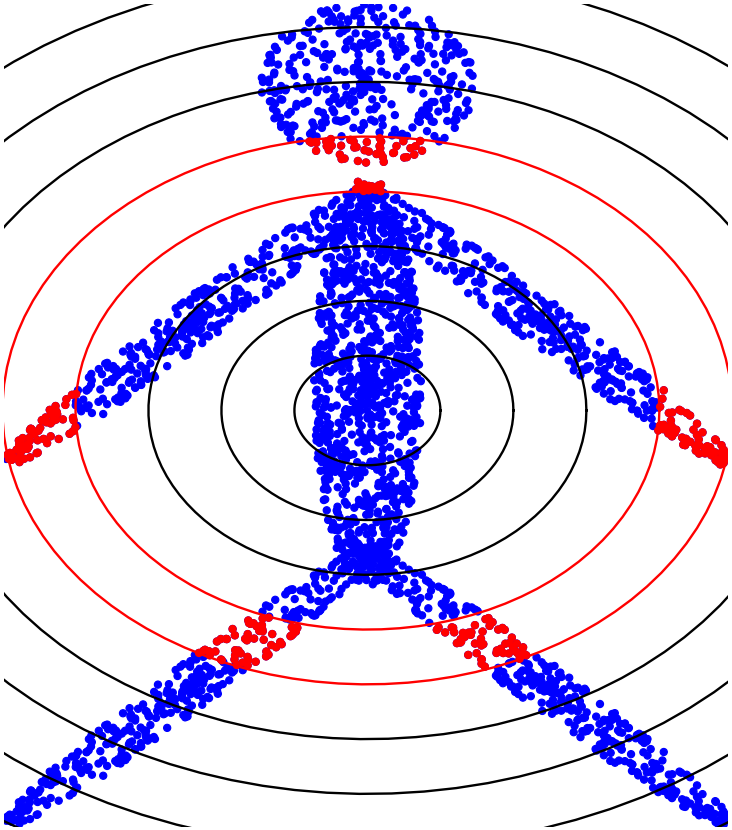

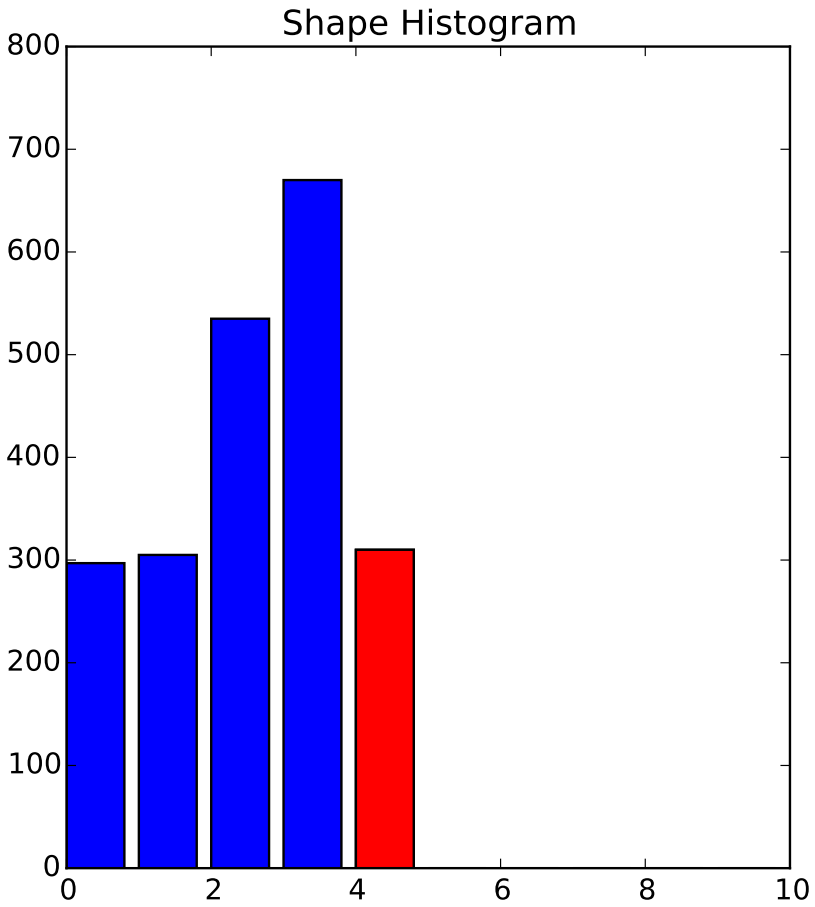

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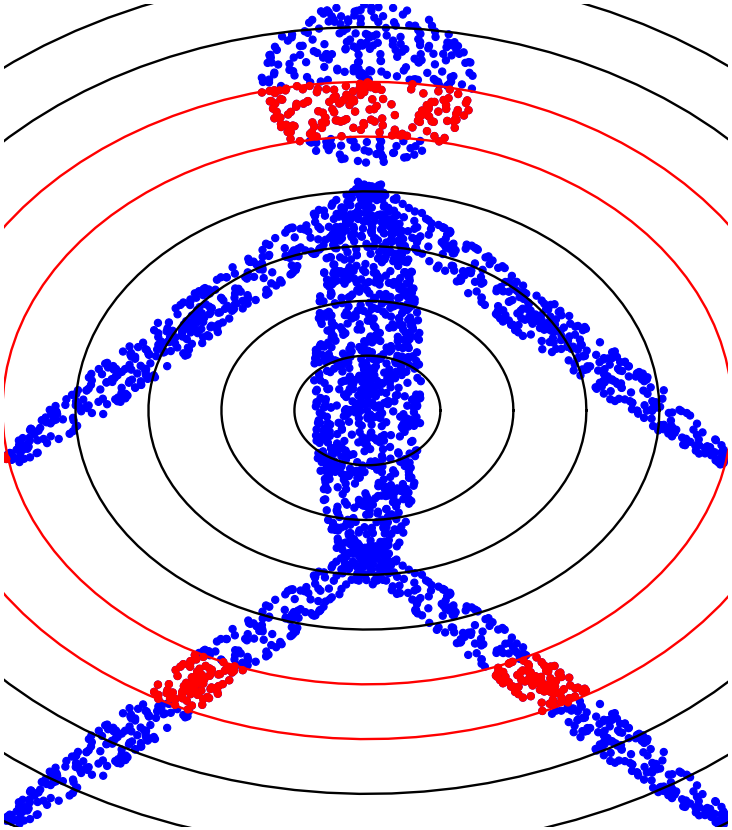

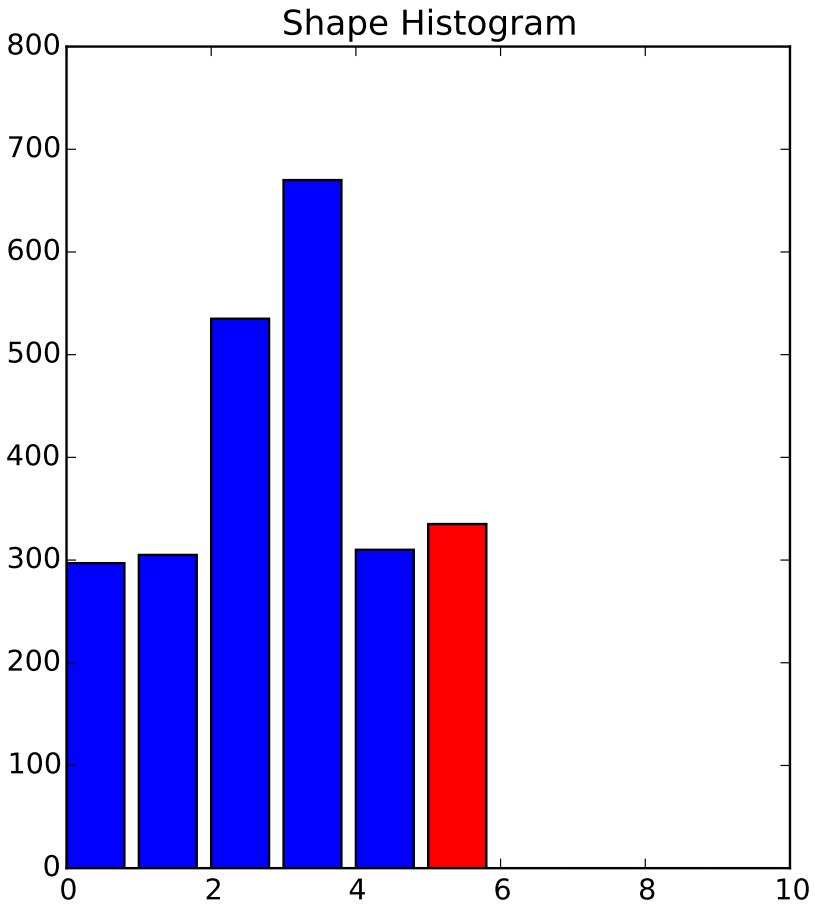

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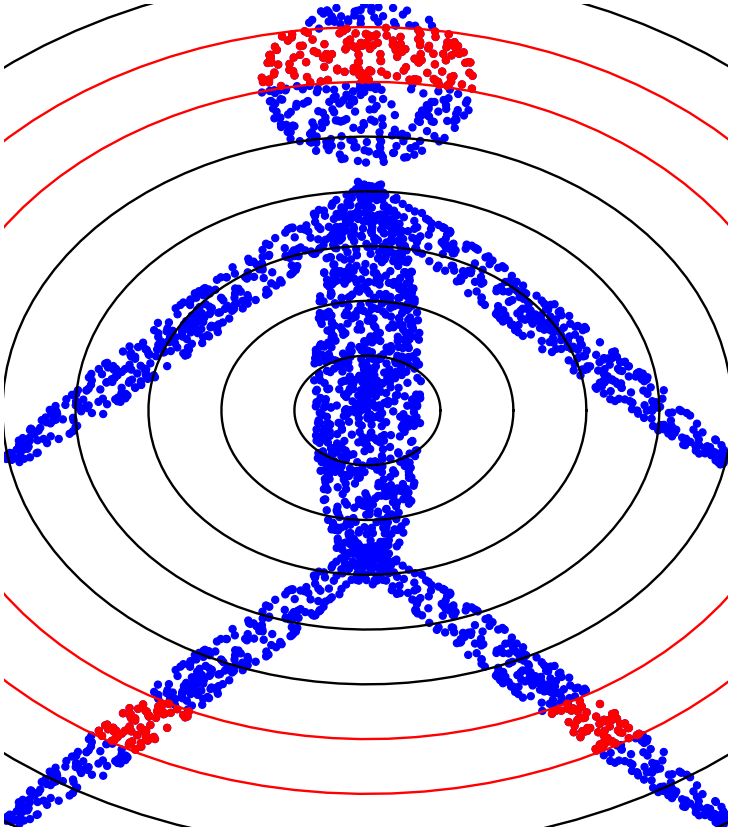

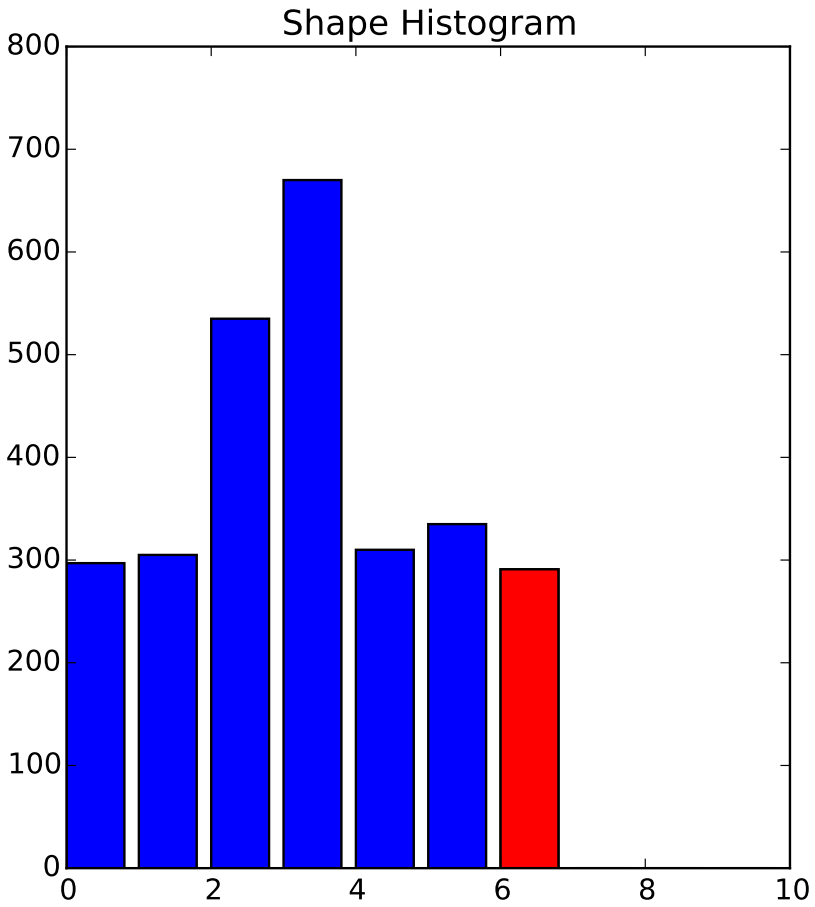

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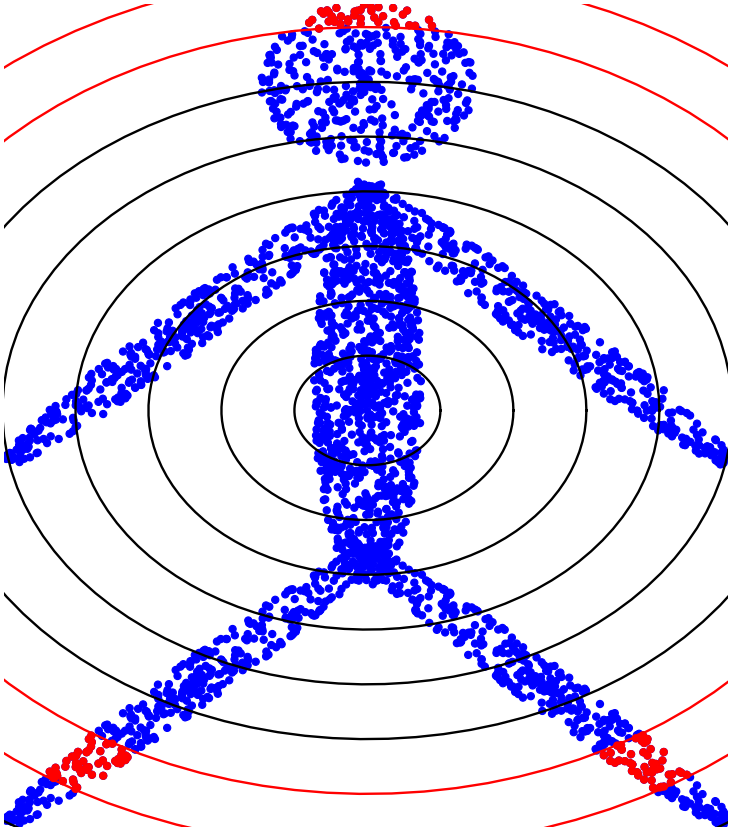

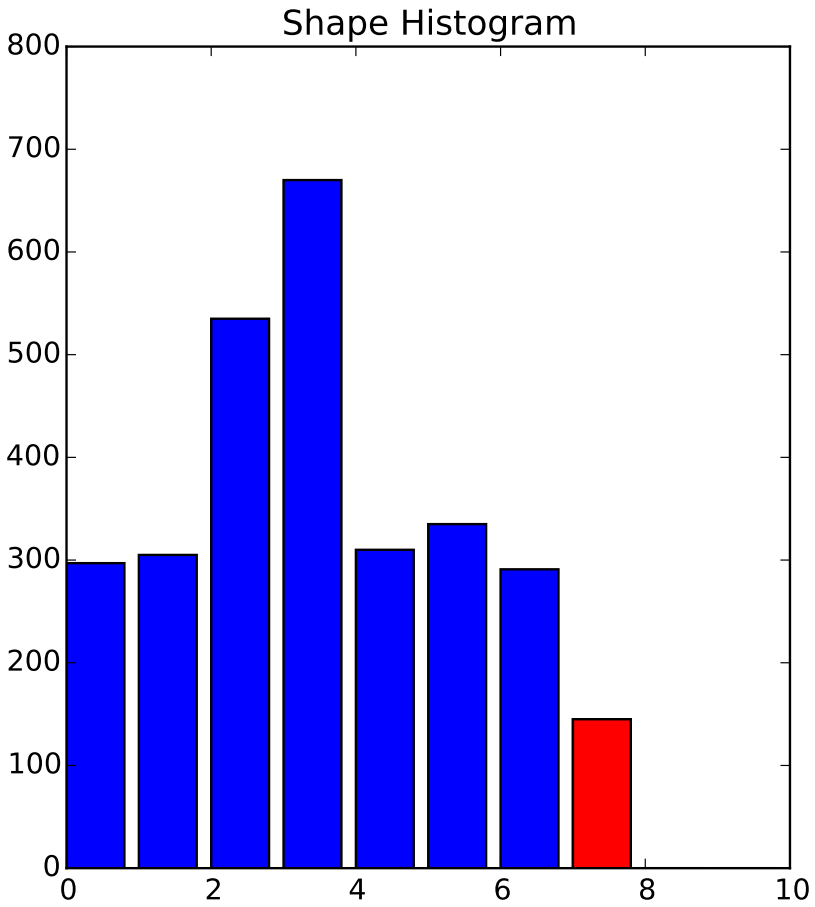

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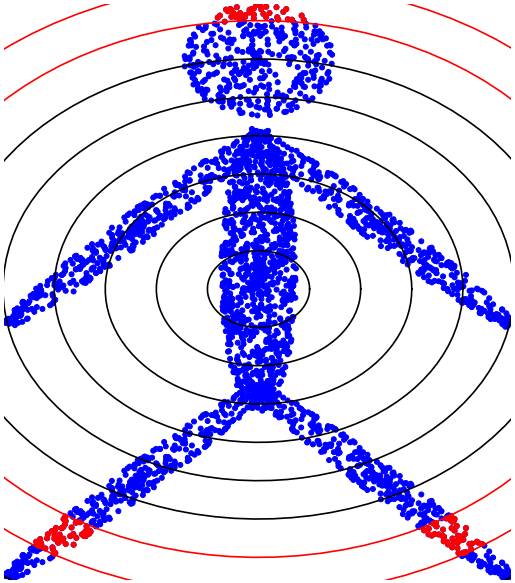
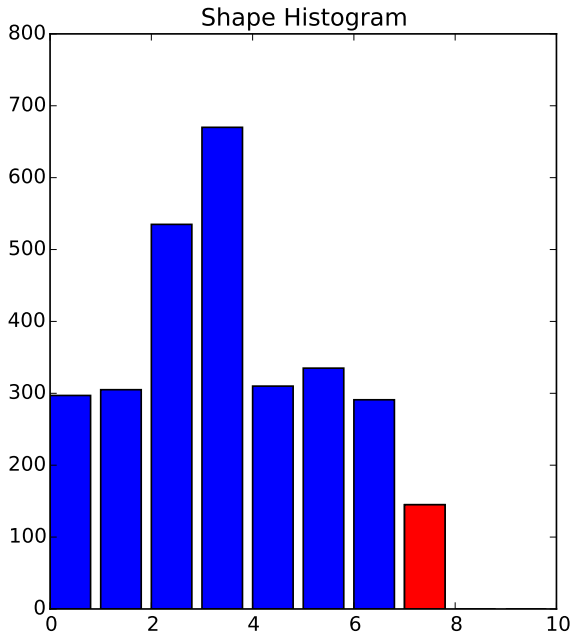
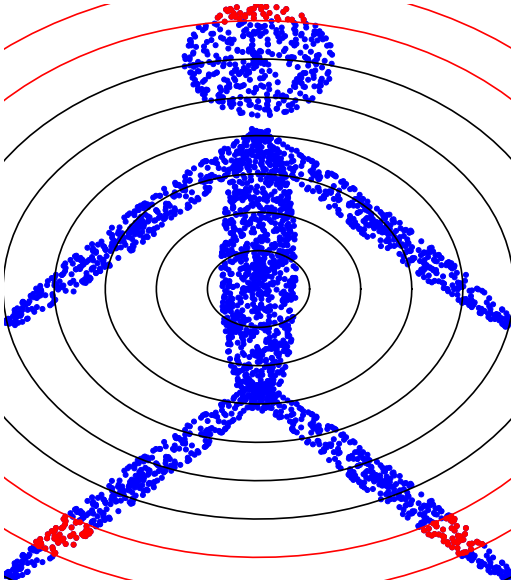
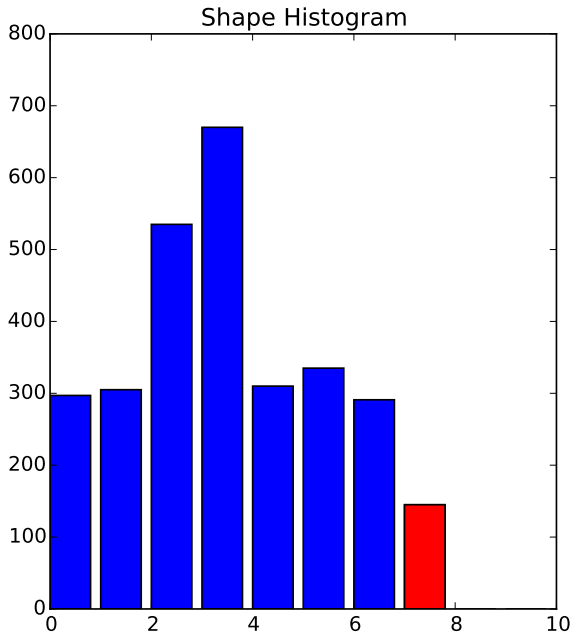




- Quick To Compute

Shape Histogram: Shells





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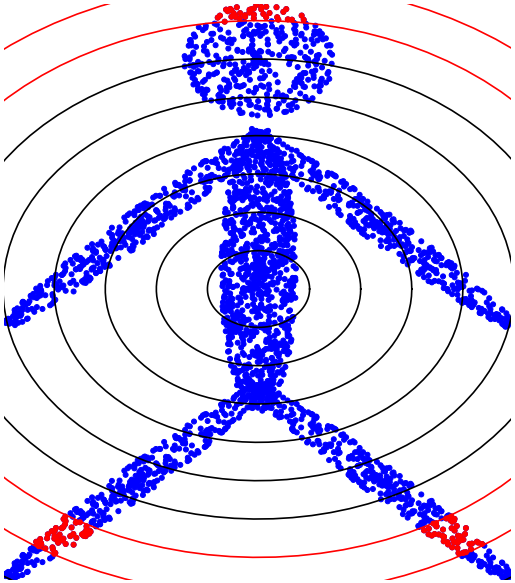
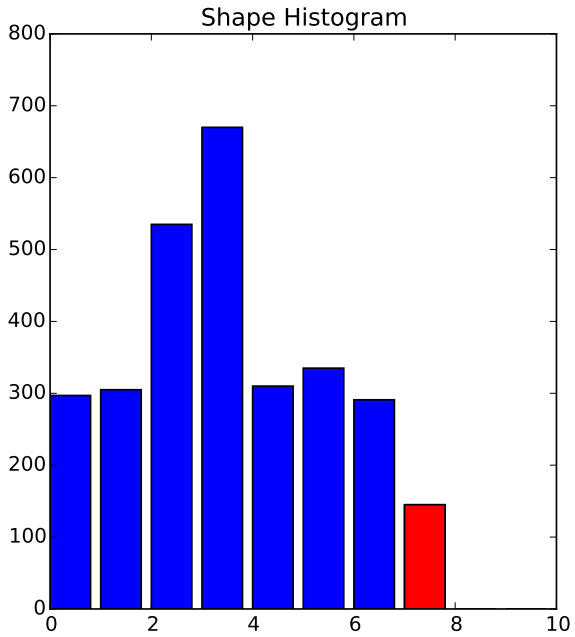
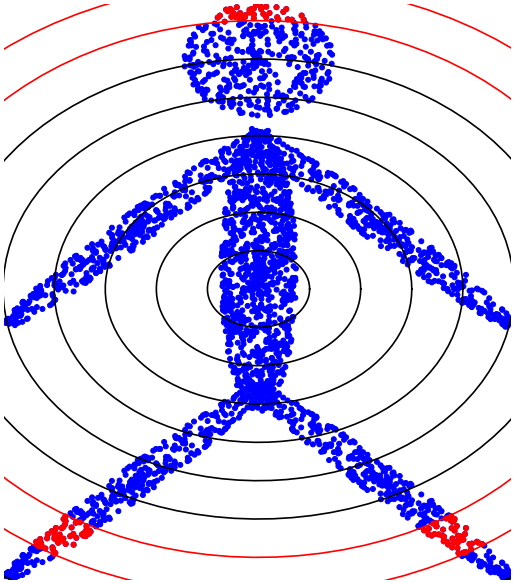
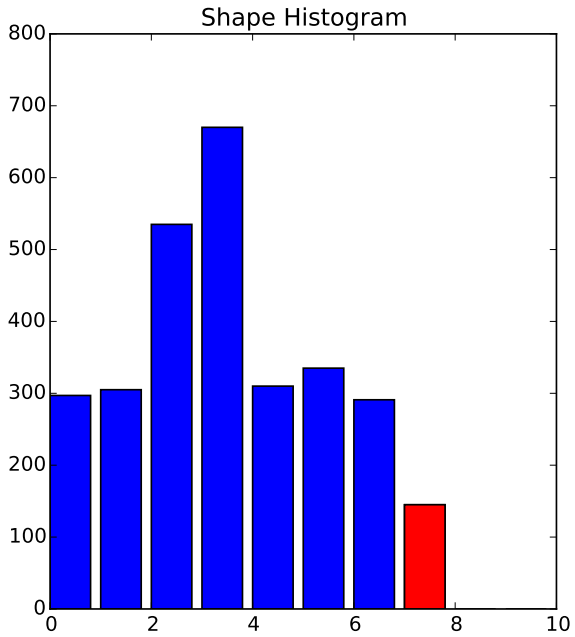




- Quick To Compute
- Concise To Store

Shape Histogram: Shells





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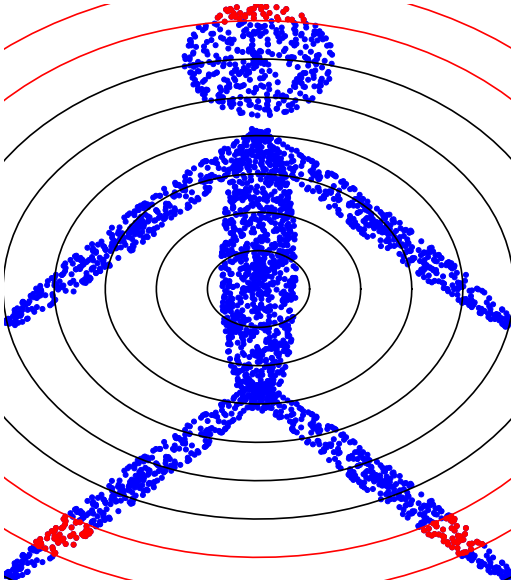
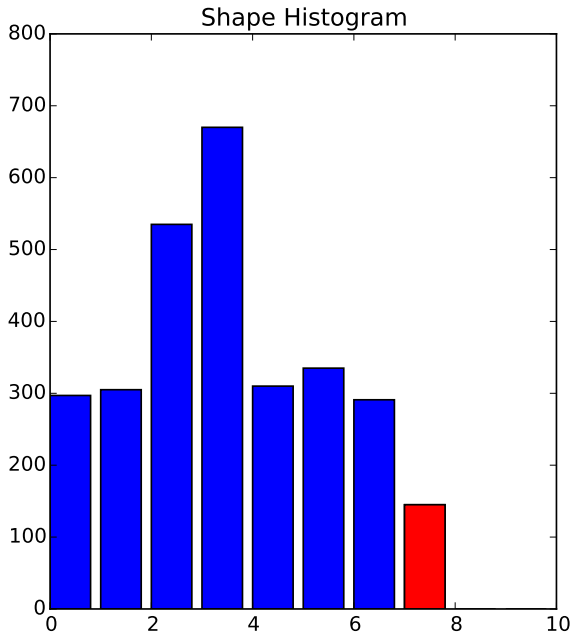
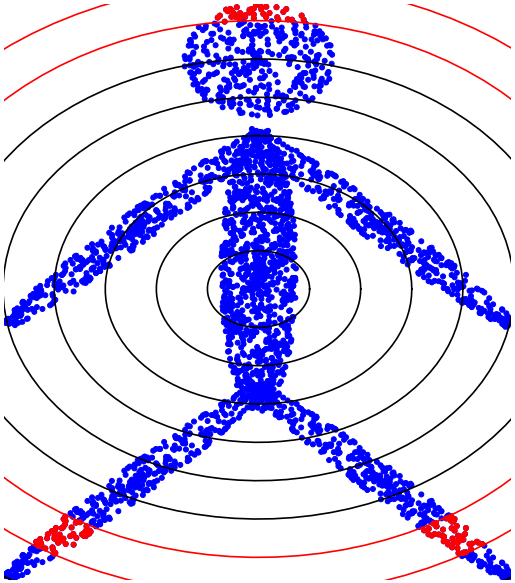
Shape Histogram: Shells

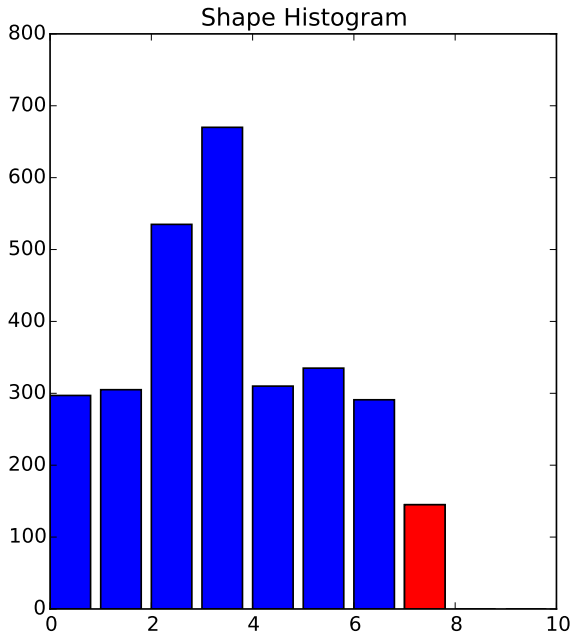




- Quick To Compute
- Concise To Store
- Rotation Invariant

Shape Histogram: Shells





- Quick To Compute
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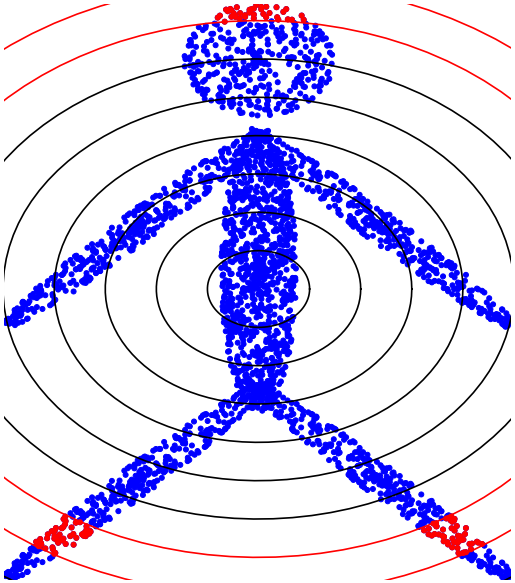
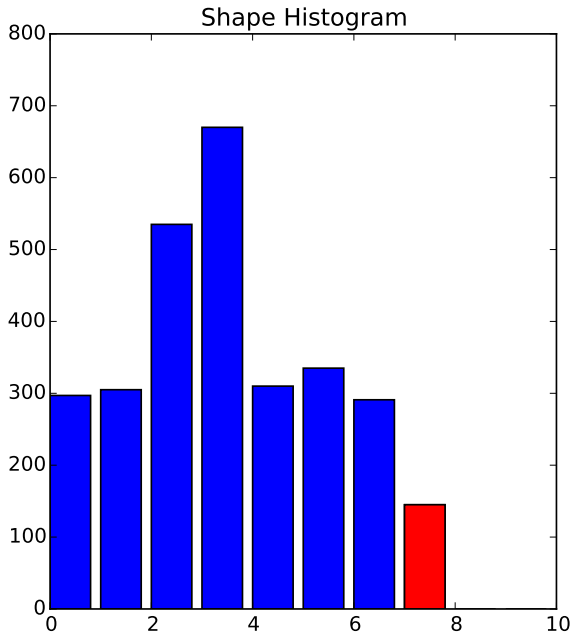
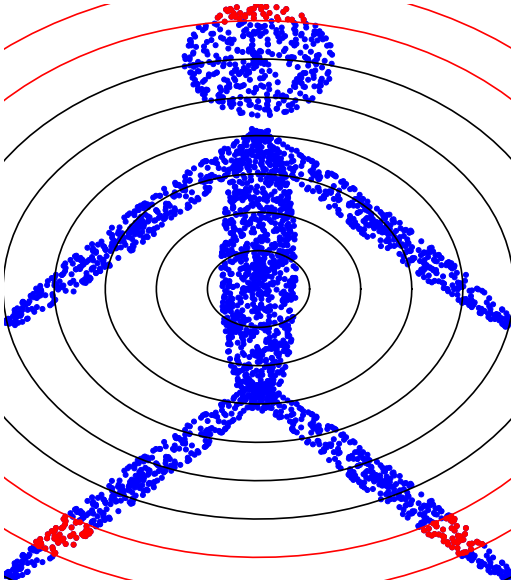
Shape Histogram: Shells

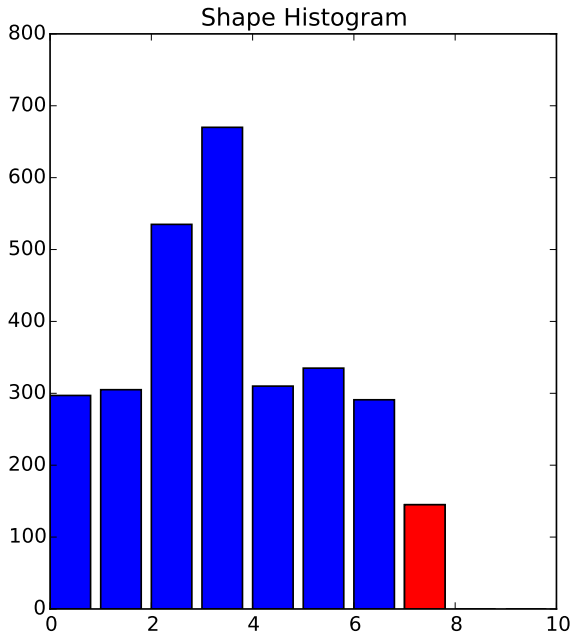




- Quick To Compute
- Concise To Store
- Rotation Invariant
- Discerning

Shape Histogram: Shells



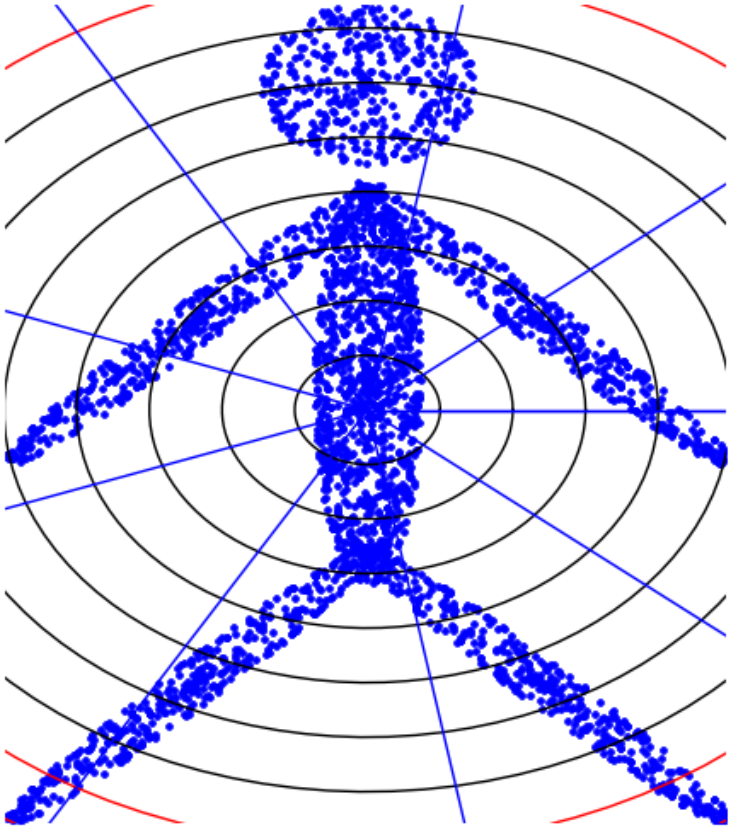


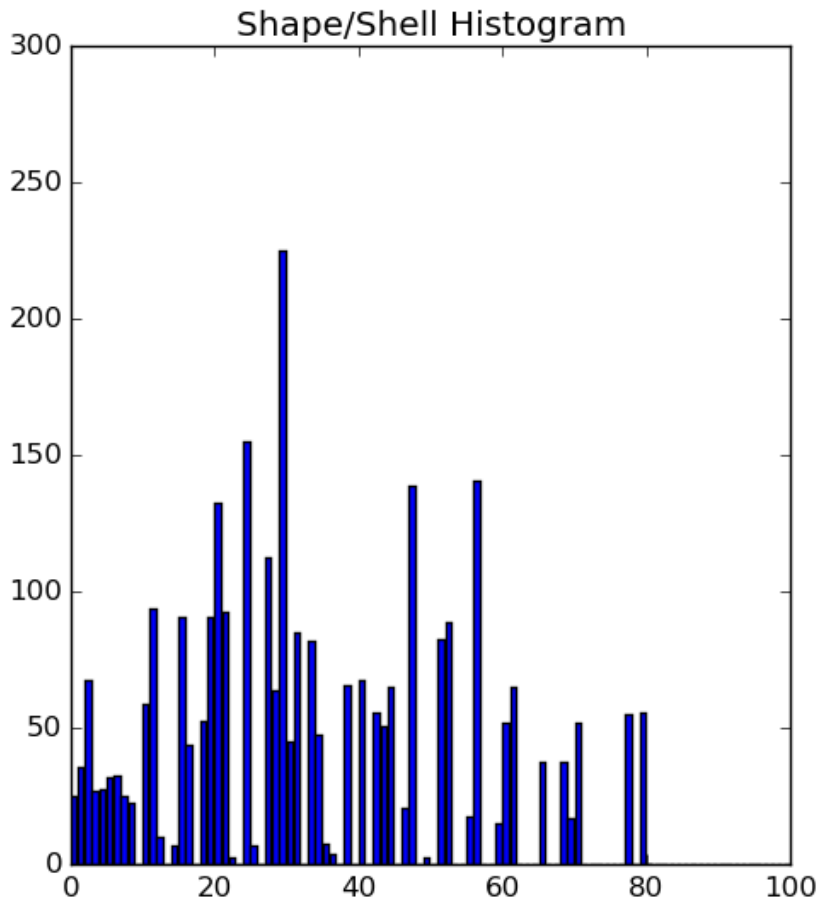
- Quick To Compute
- Concise To Store
- Rotation Invariant
- Discerning

Shape Histogram: Shells

What can't it tell apart?
SHOW VIDEO

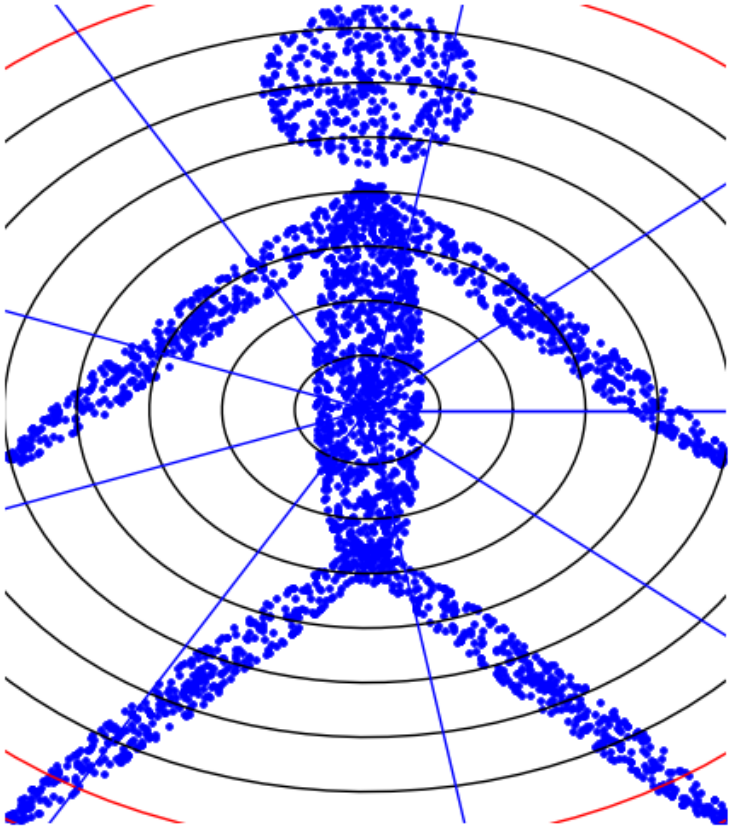
Shape Histogram: Shells And Sectors

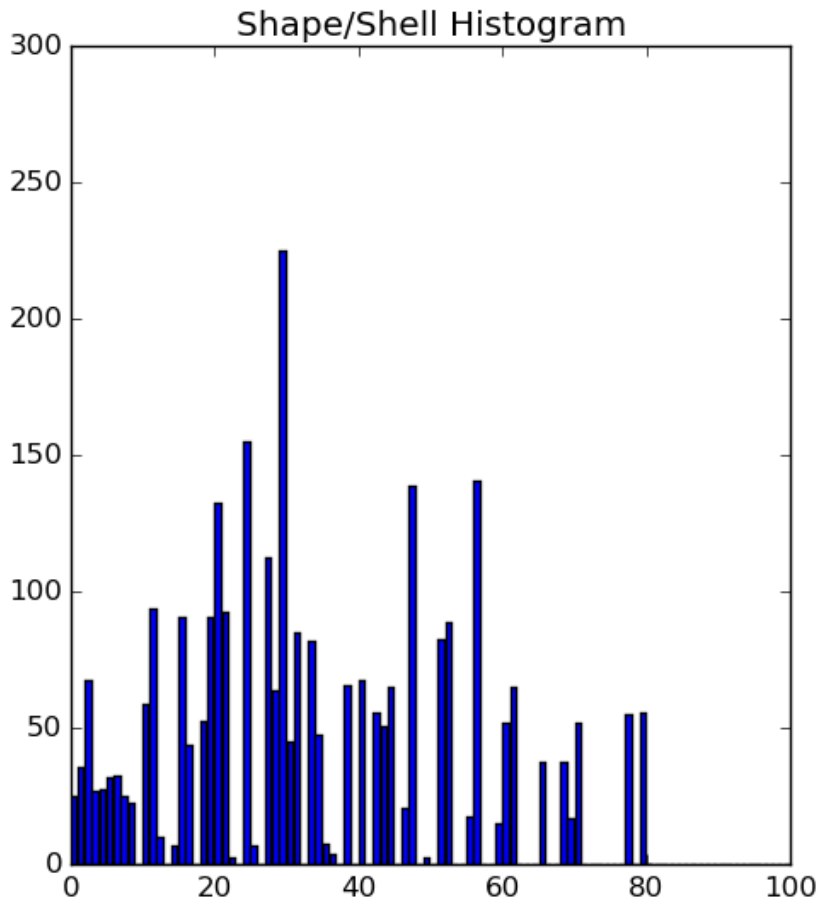




SHOW VIDEO

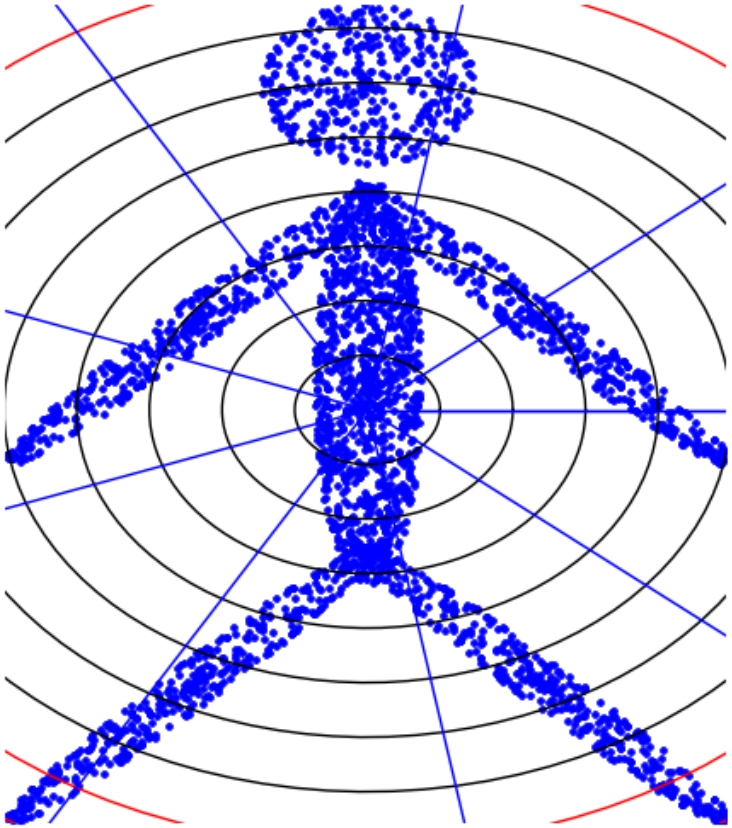
Shape Histogram: Shells And Sectors

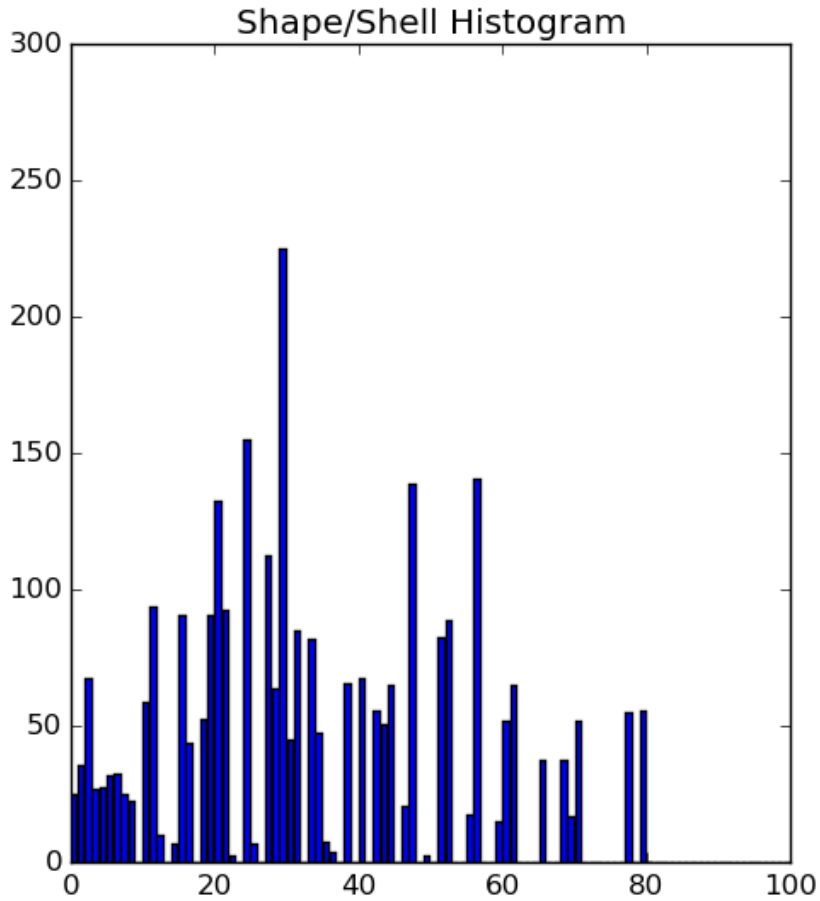




Still Rotation Invariant?

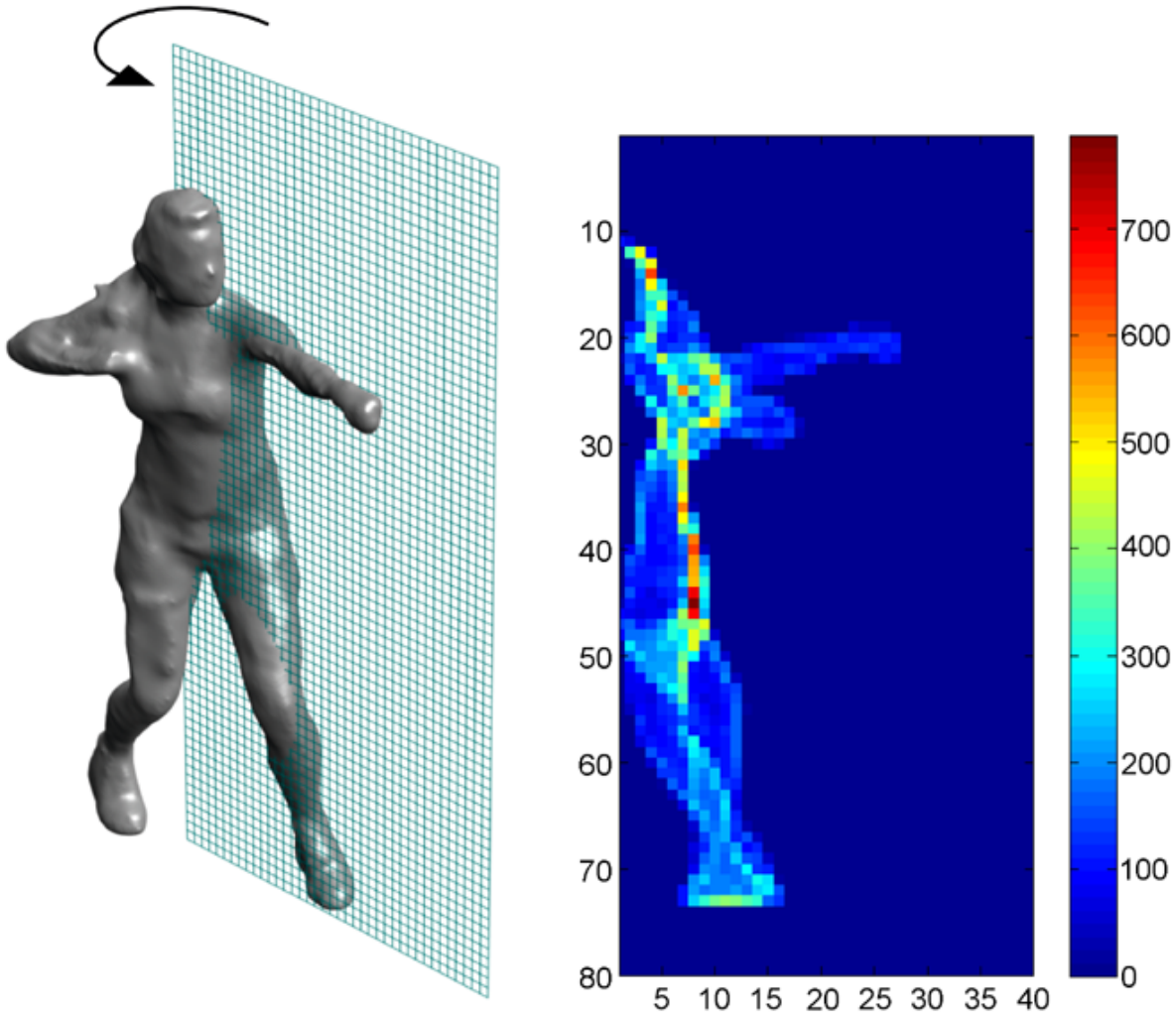
Shape Histogram: Shells And Sectors





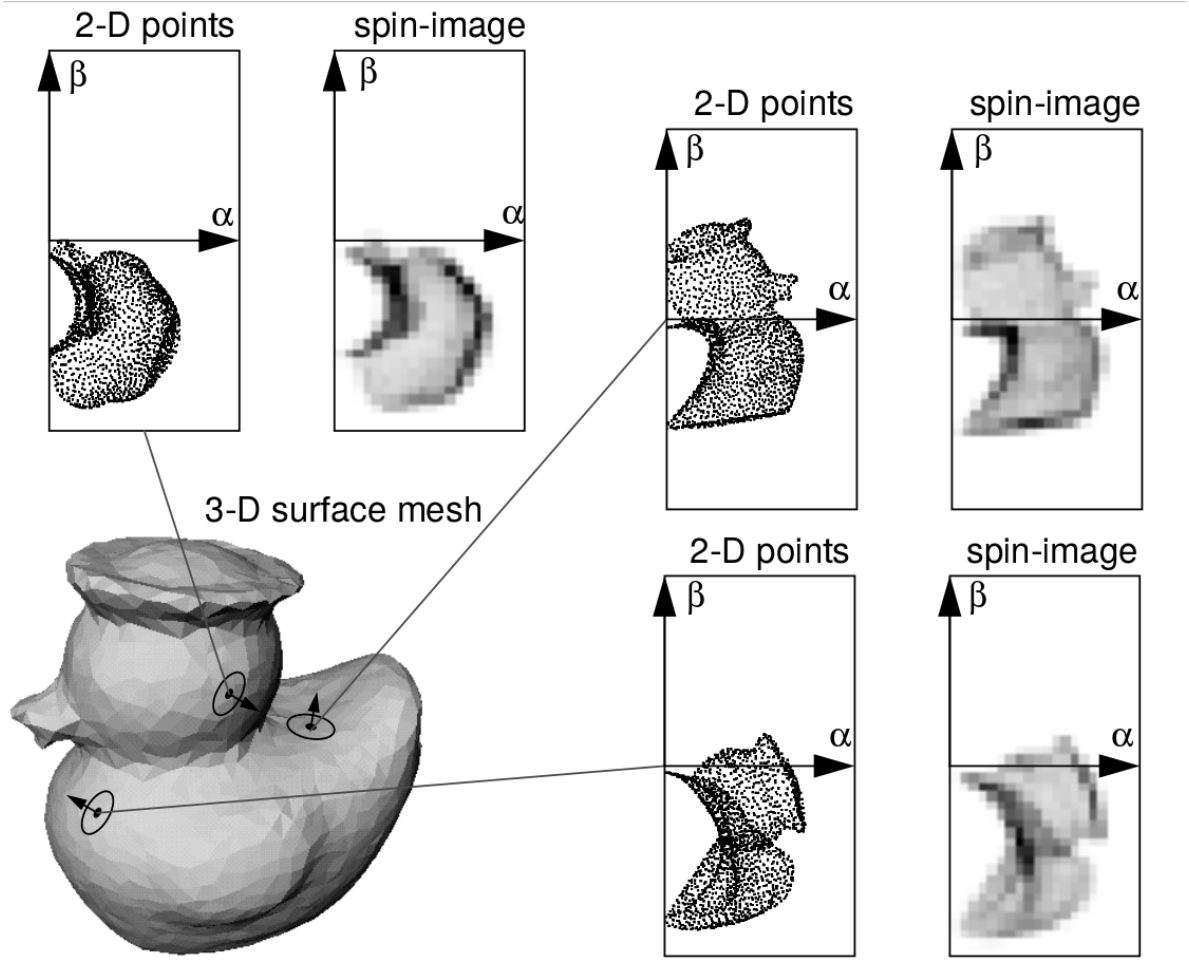
- Sort sectors within each shell
- Record PCA *eigenvalues* within each shell

Spin Images



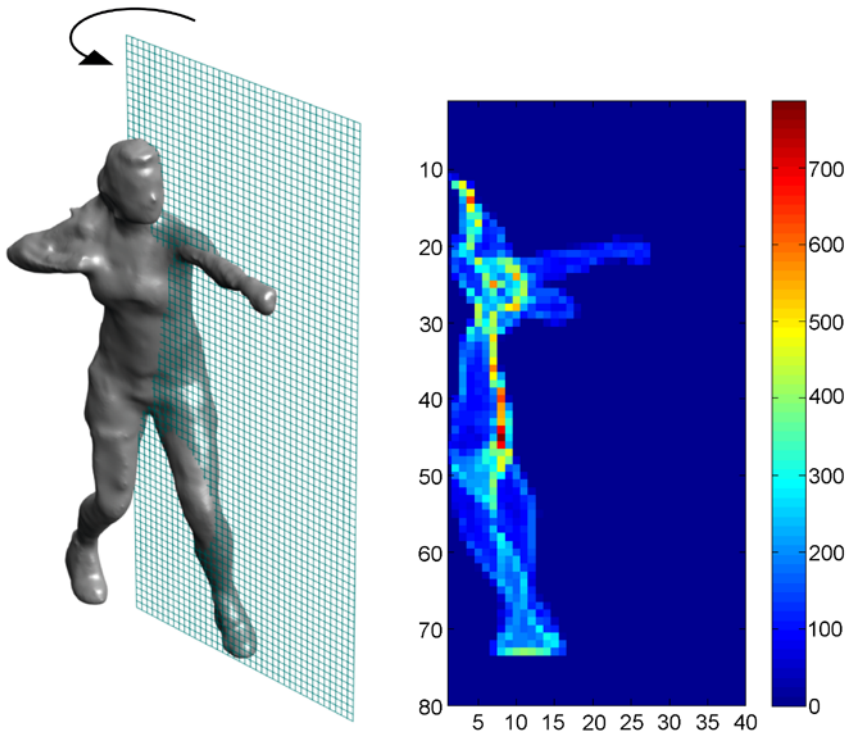
Johnson/Herbert 1999, Figure Huang 2010

Spin Images: Rubber Duck



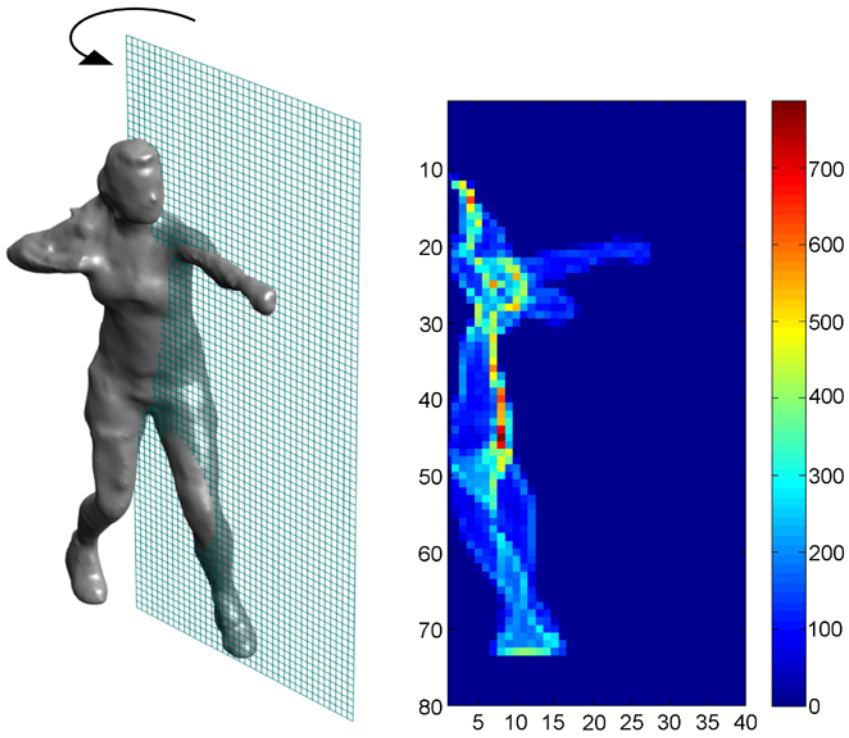
Johnson/Herbert 1999

Spin Images



- Quick To Compute

Spin Images



- Quick To Compute

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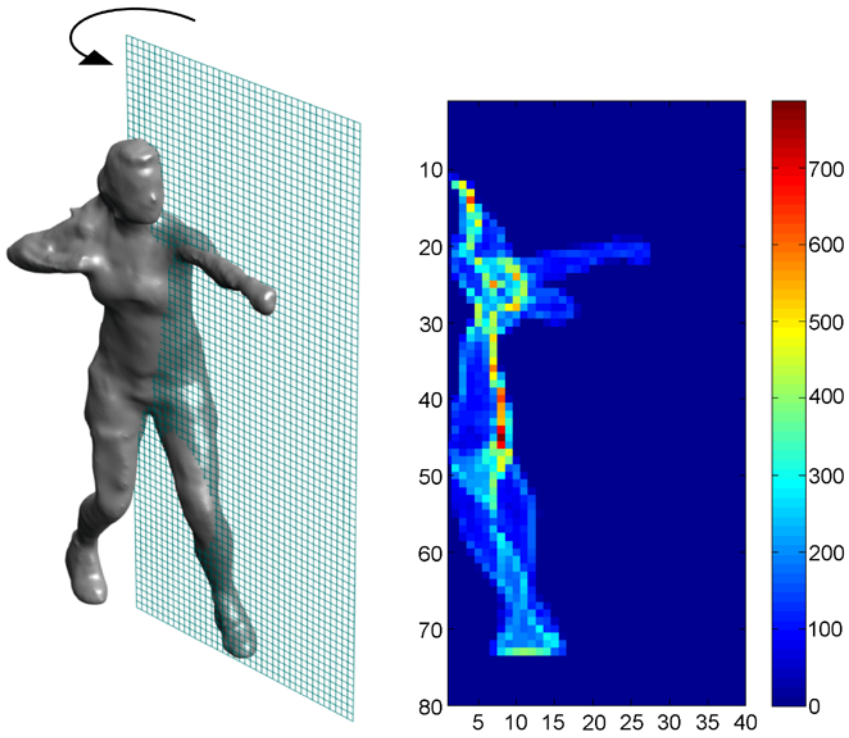
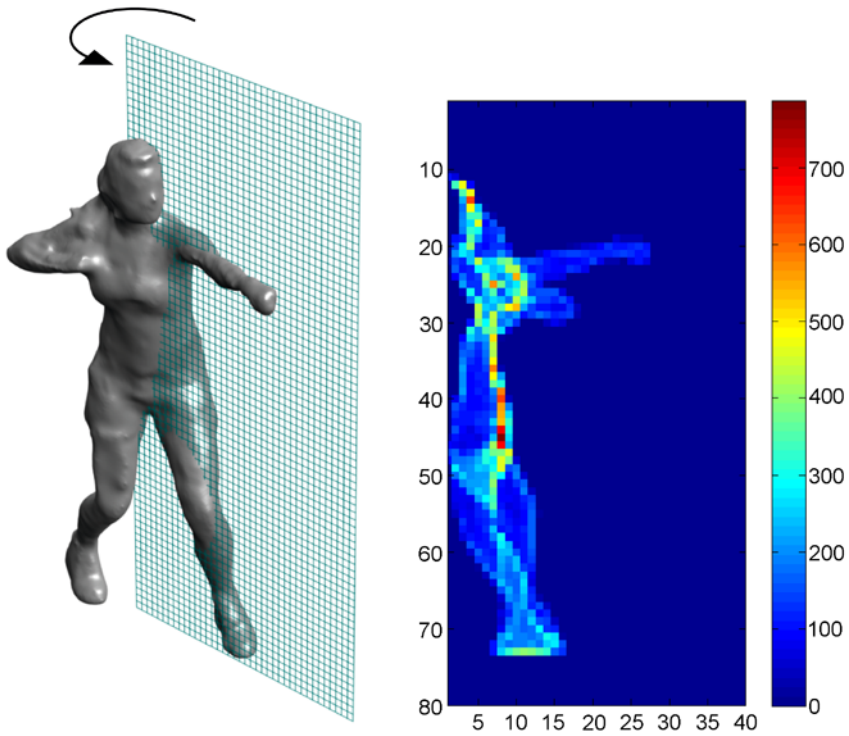


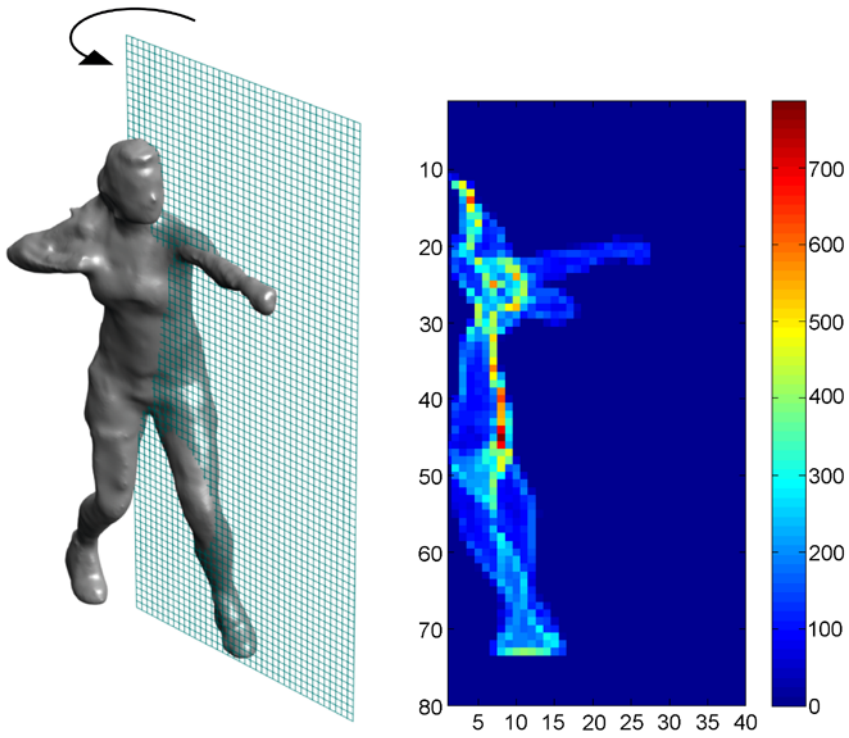
- Quick To Compute
- Concise To Store

Spin Images



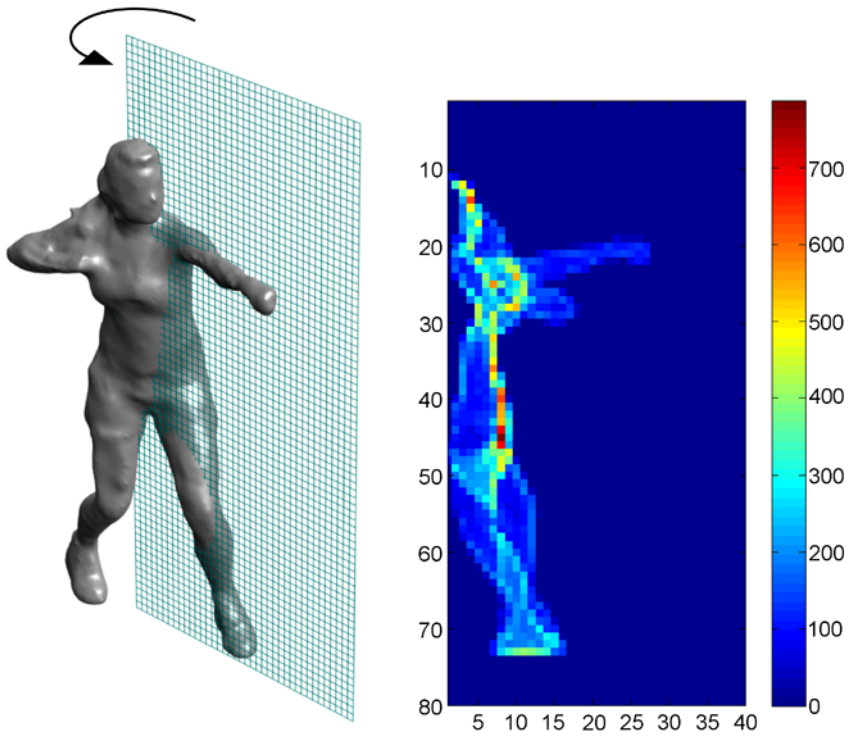
- ▷ Quick To Compute
- ▷ Concise To Store (Can compress images)

Spin Images



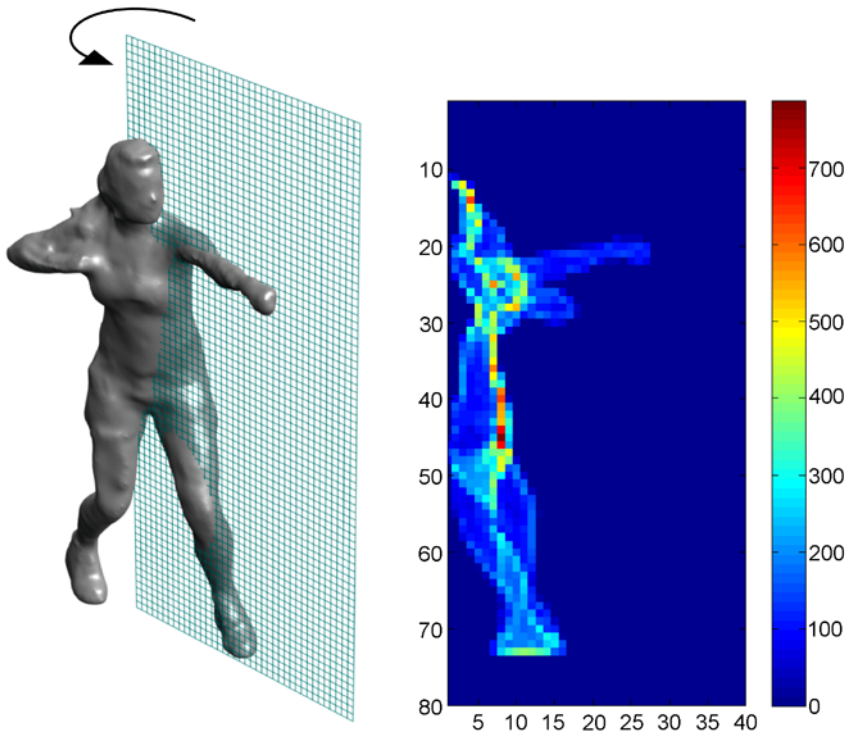
- ▷ Quick To Compute
- ▷ Concise To Store (Can compress images)
- ▷ Rotation Invariant

Spin Images



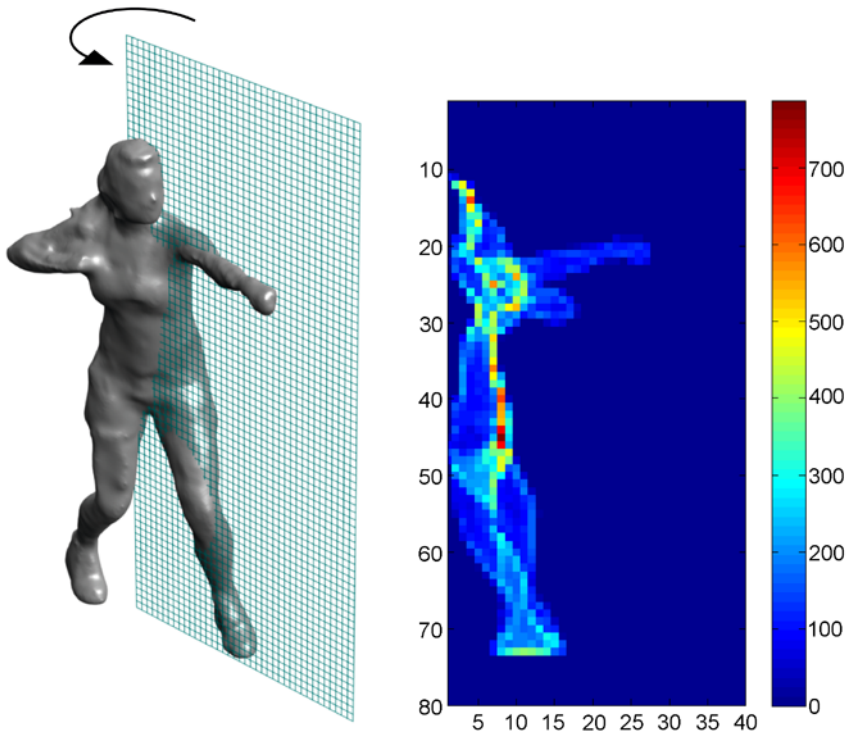
- ▷ Quick To Compute
- ▷ Concise To Store (Can compress images)
- ▷ Rotation Invariant (Careful with principal axis stability)

Spin Images



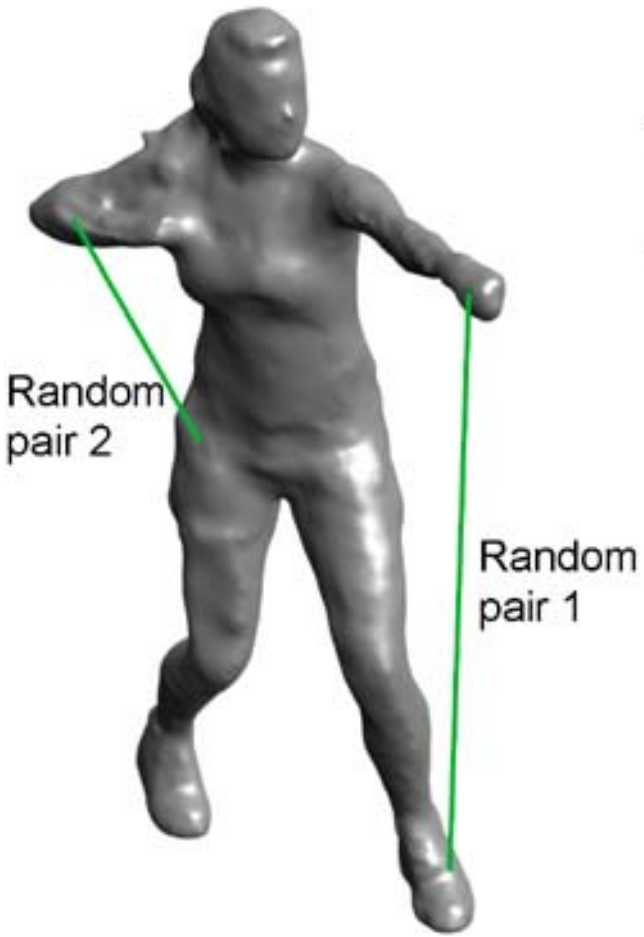
- ▷ Quick To Compute
- ▷ Concise To Store (Can compress images)
- ▷ Rotation Invariant (Careful with principal axis stability)
- ▷ Discerning

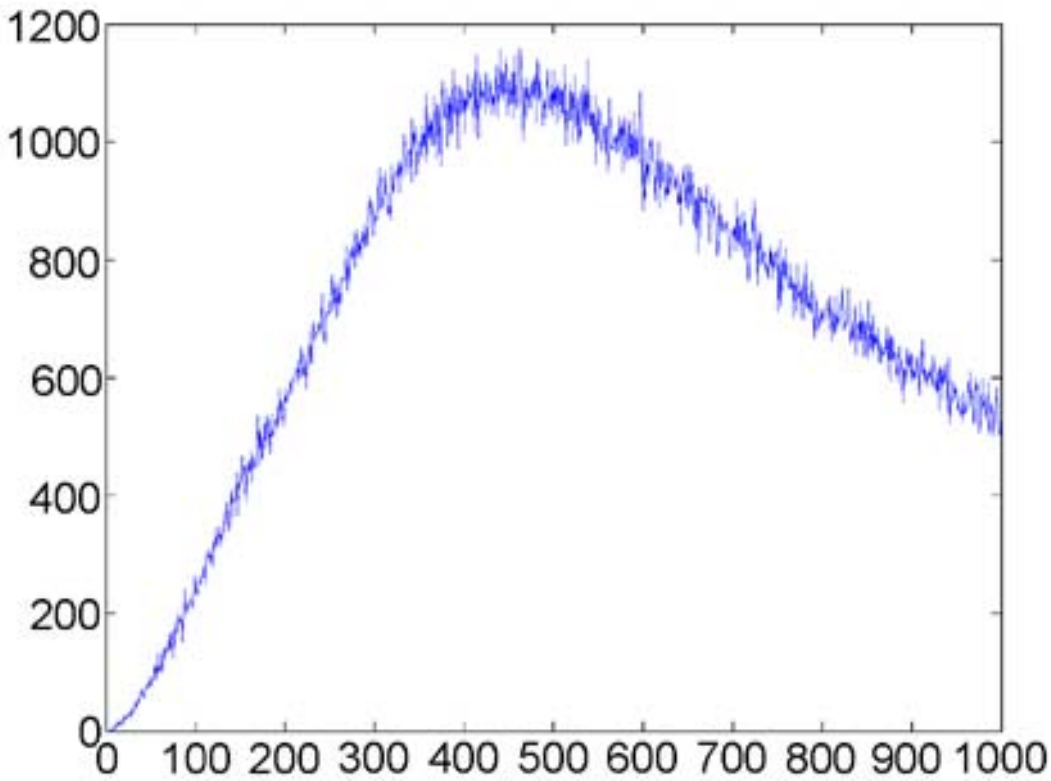
Spin Images



- ▷ Quick To Compute
- ▷ Concise To Store (Can compress images)
- ▷ Rotation Invariant (Careful with principal axis stability)
- ▷ Discerning

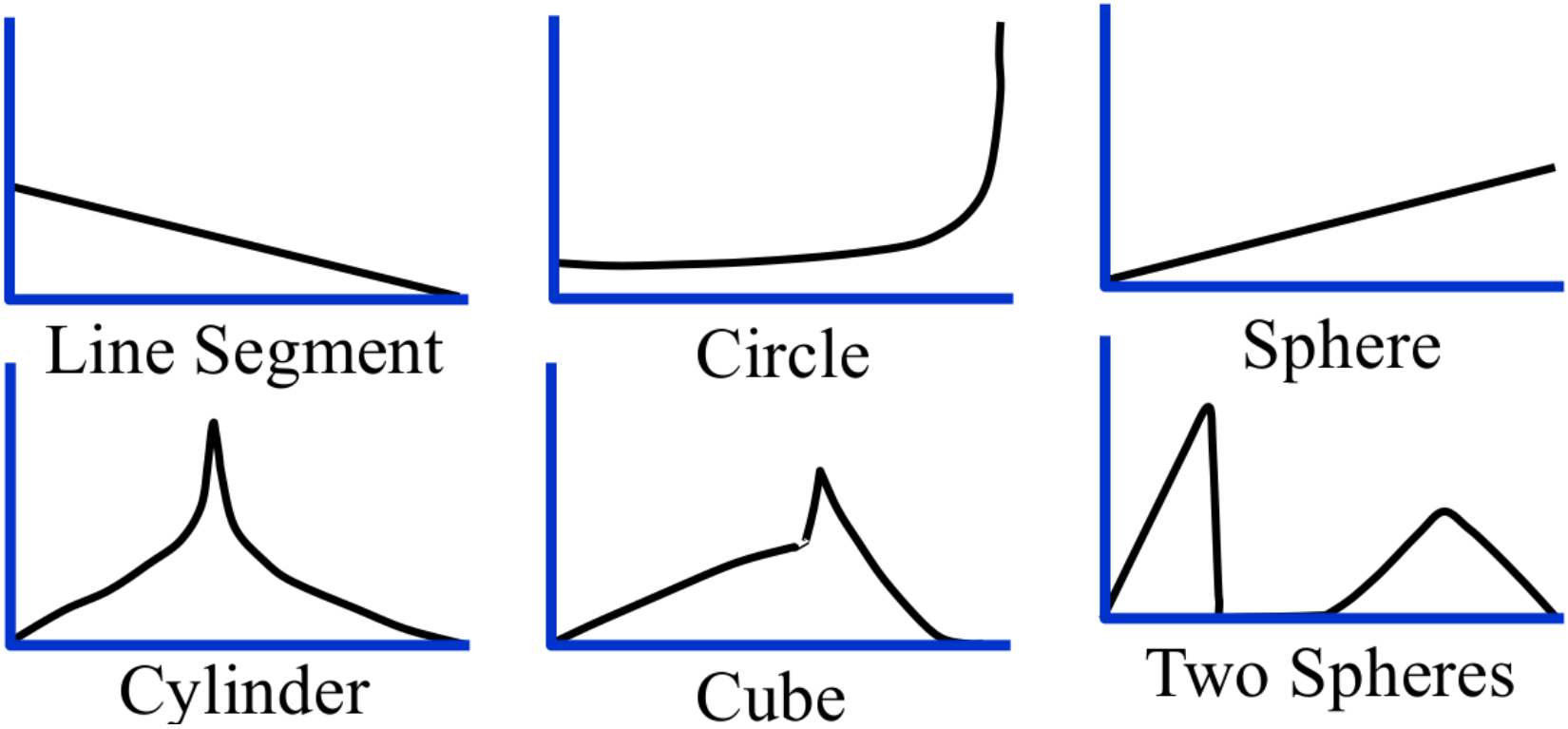
D2: Distance Histograms





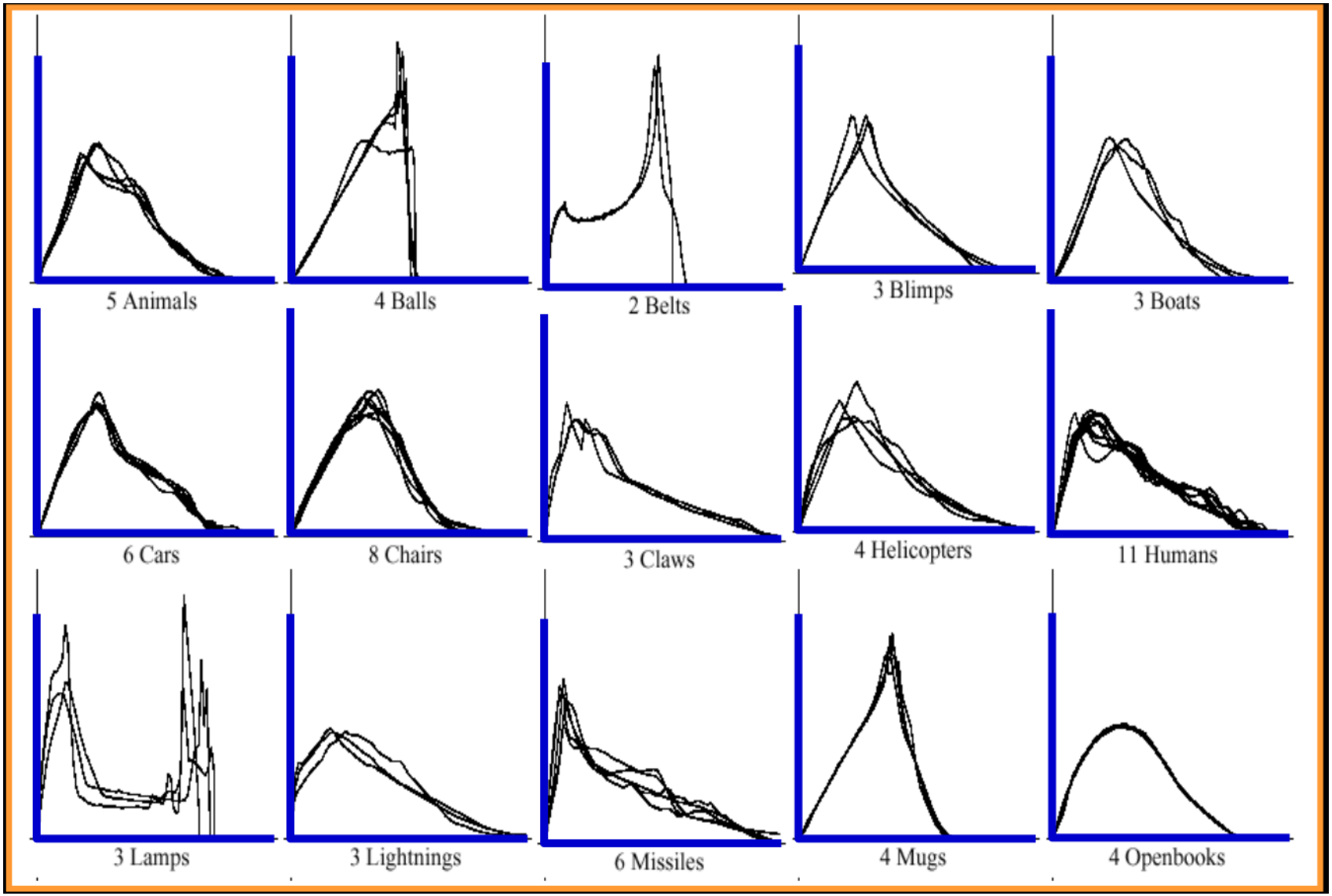
Osada 2003, Figure from Huang 2010

D2: Primitive Examples



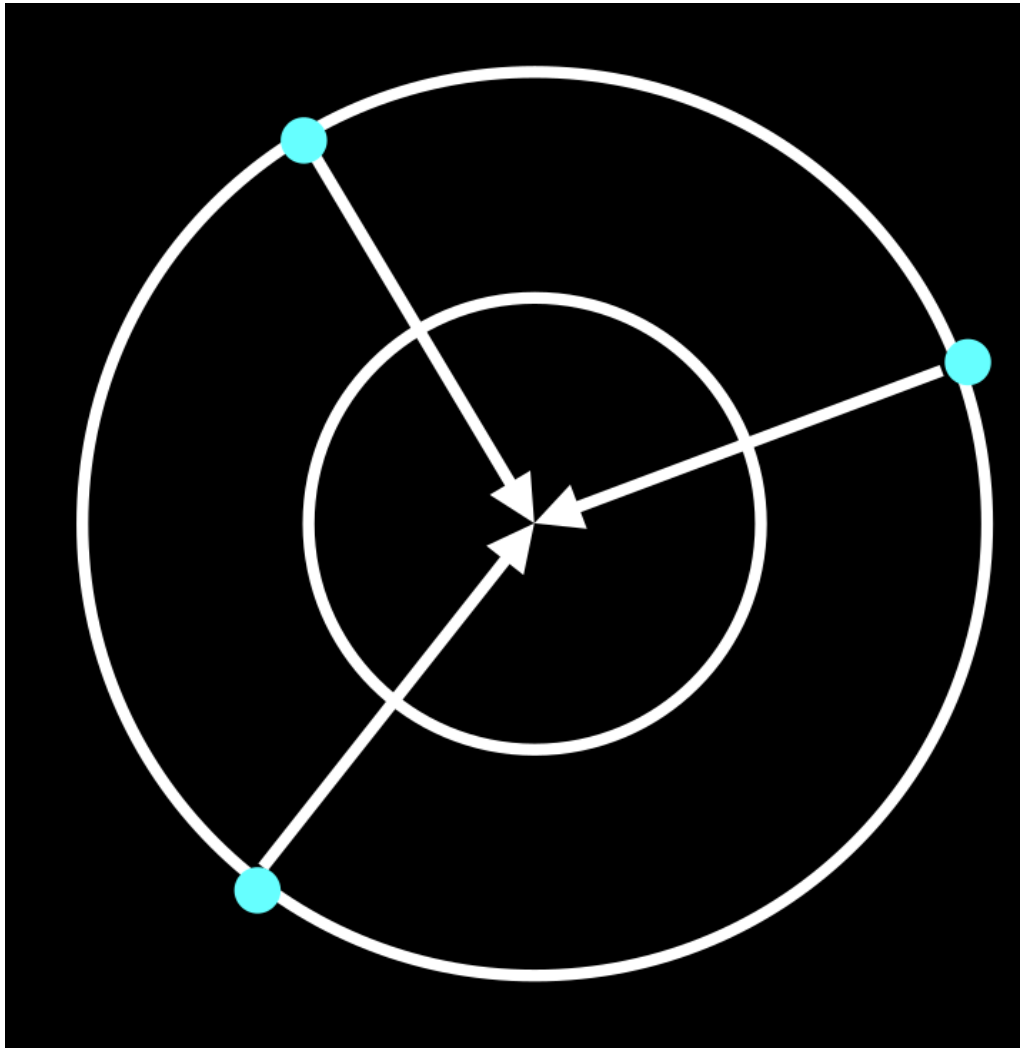
Osada 2003

D2: Real Examples



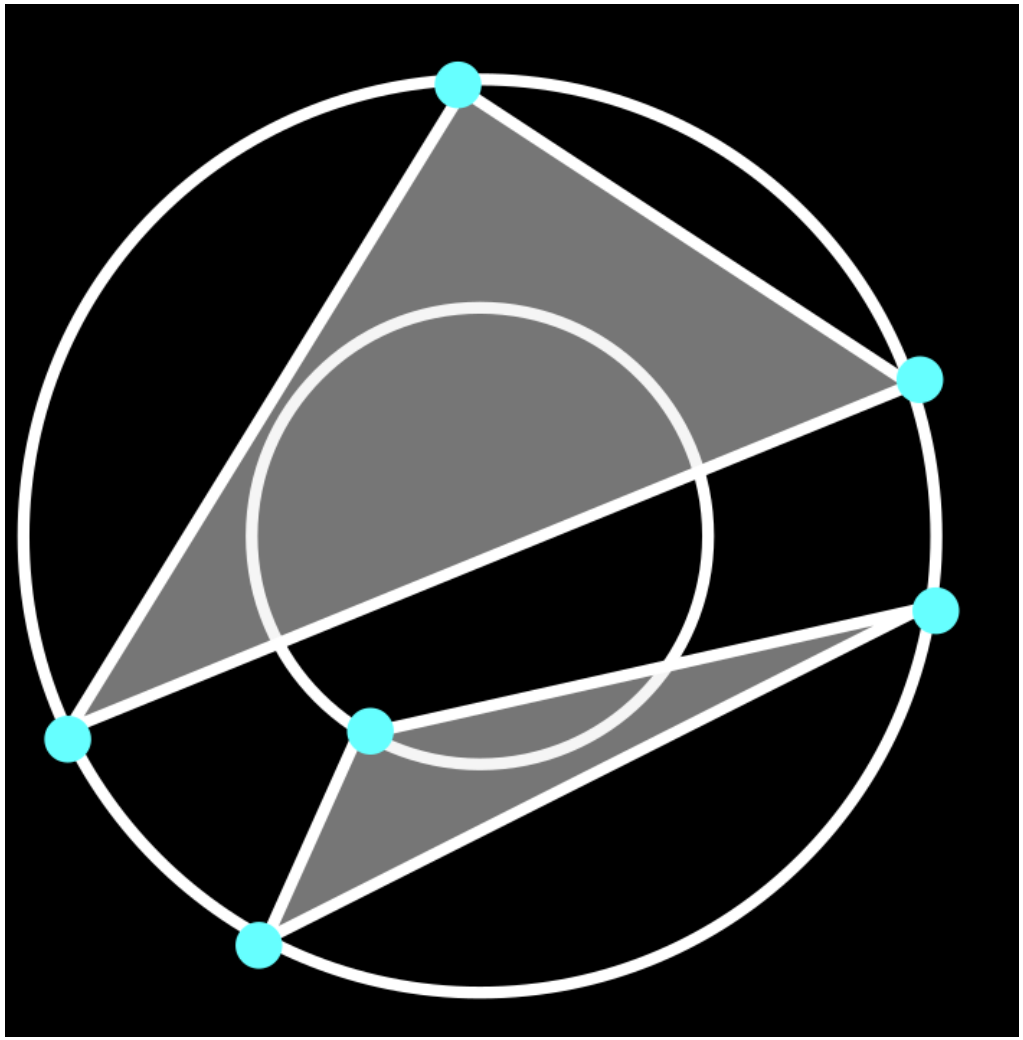
Osada 2003

D1: Randomly Sample Points



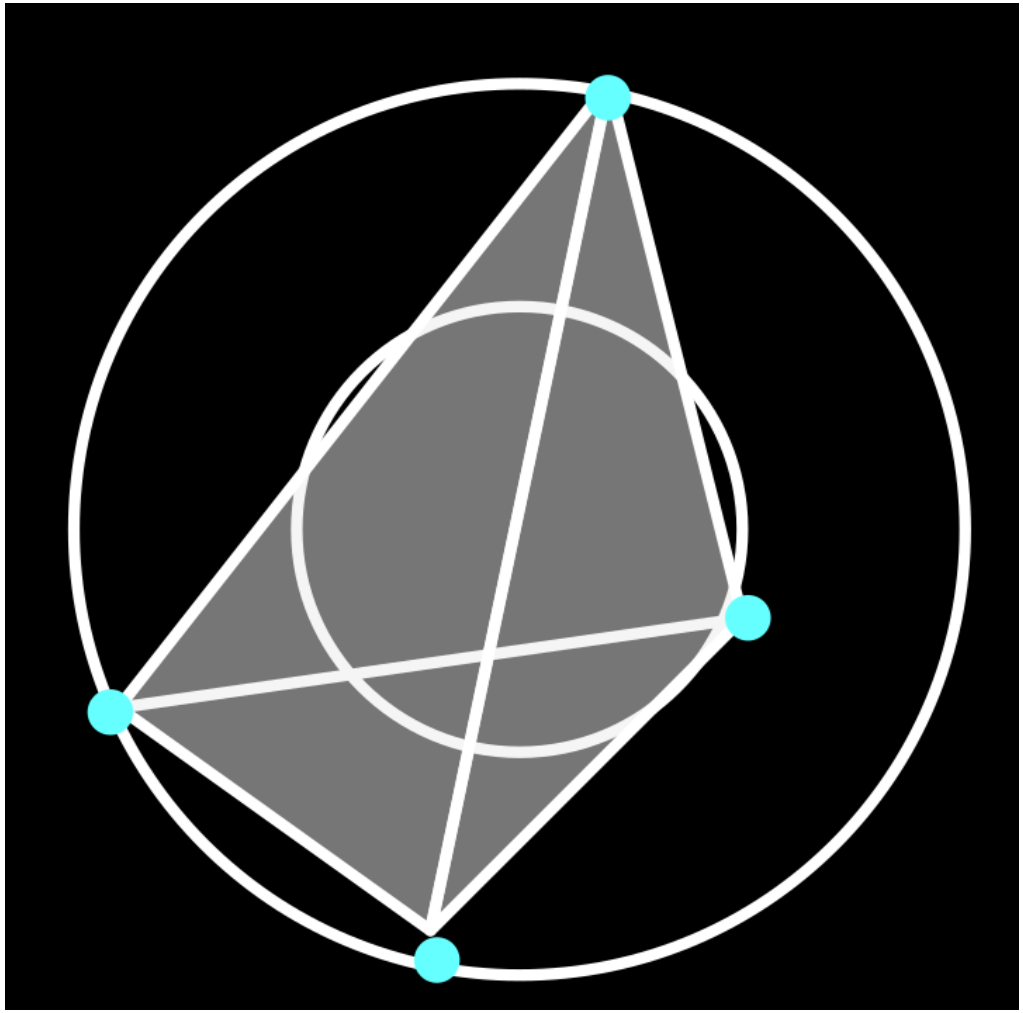
Osada 2003

D3: Randomly Sample Areas



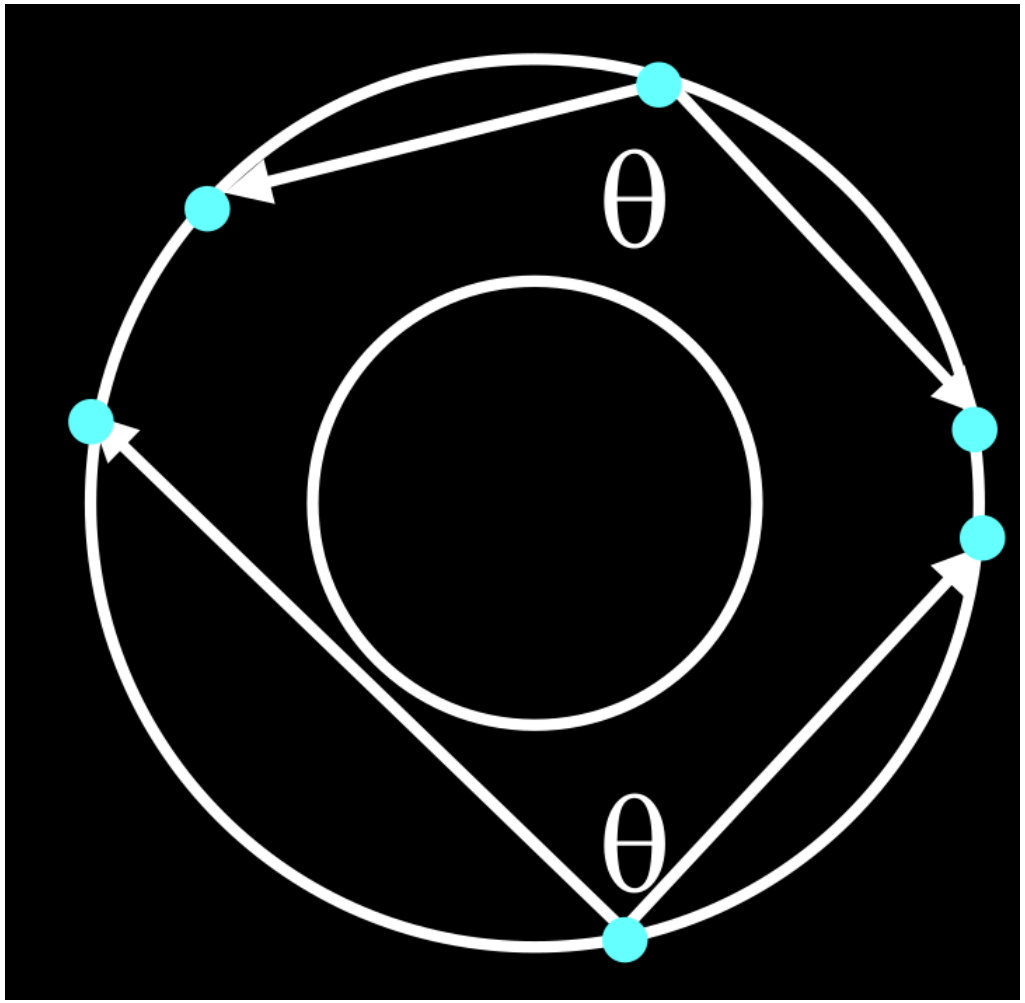
Osada 2003

D4: Randomly Sample Volumes



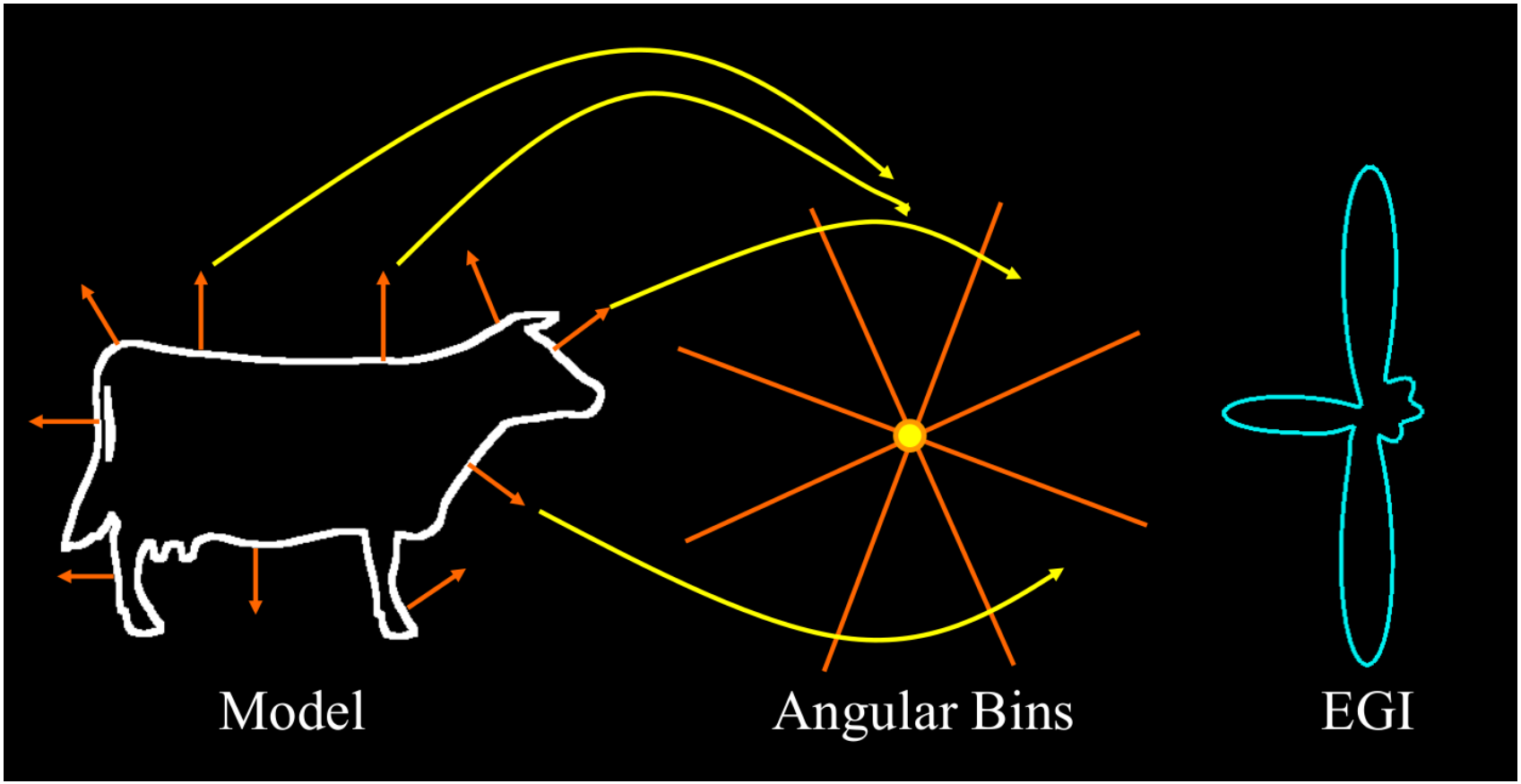
Osada 2003

A3: Randomly Sample Angles



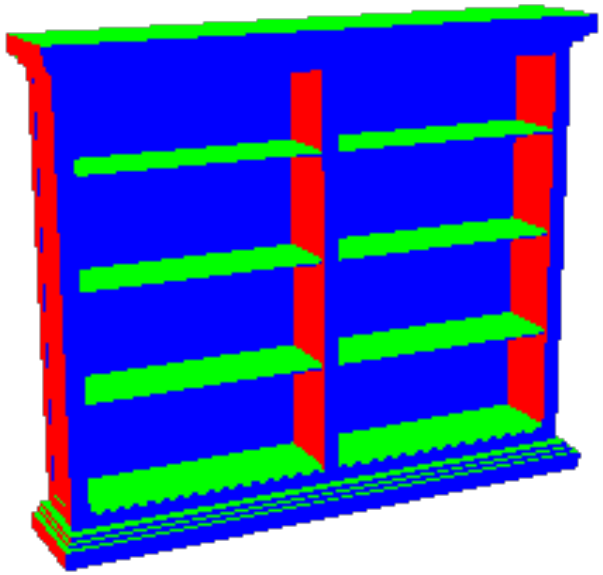
Osada 2003

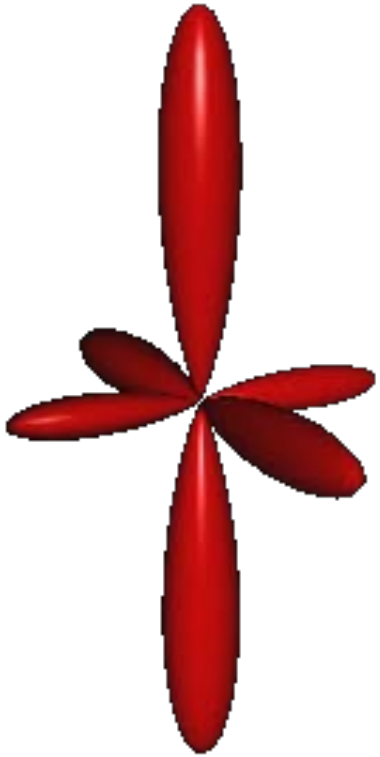
Extended Gaussian Image



Funkhouser 2004

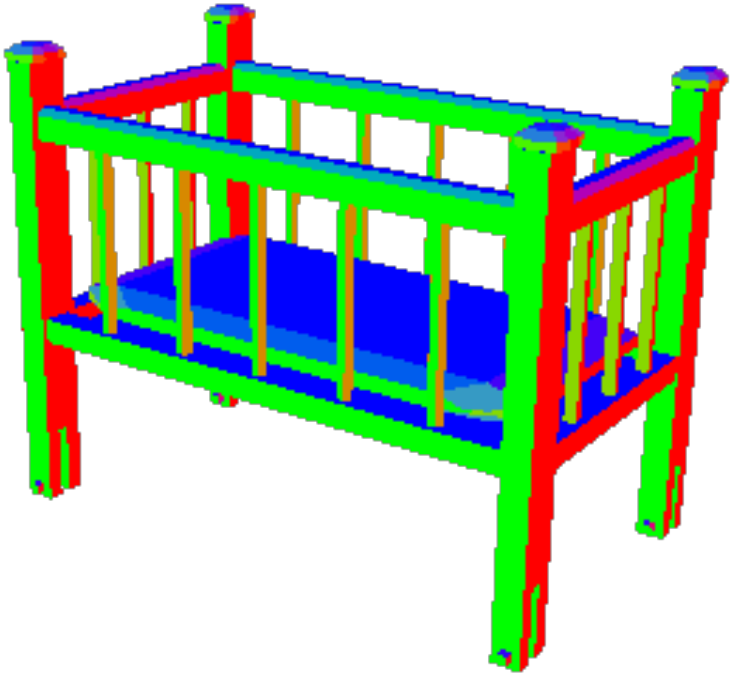
Extended Gaussian Image

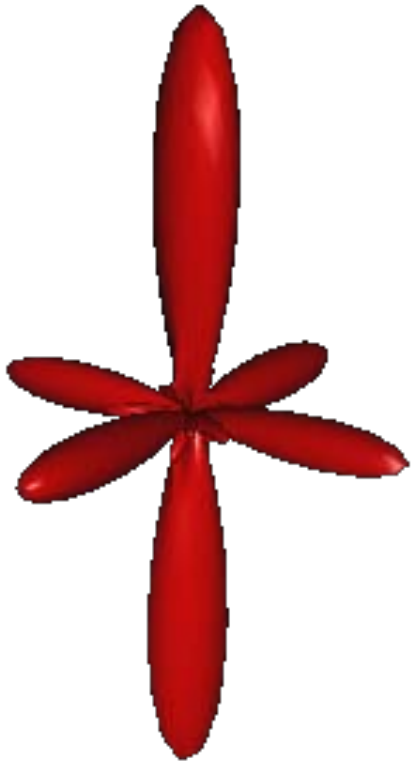




Funkhouser 2004

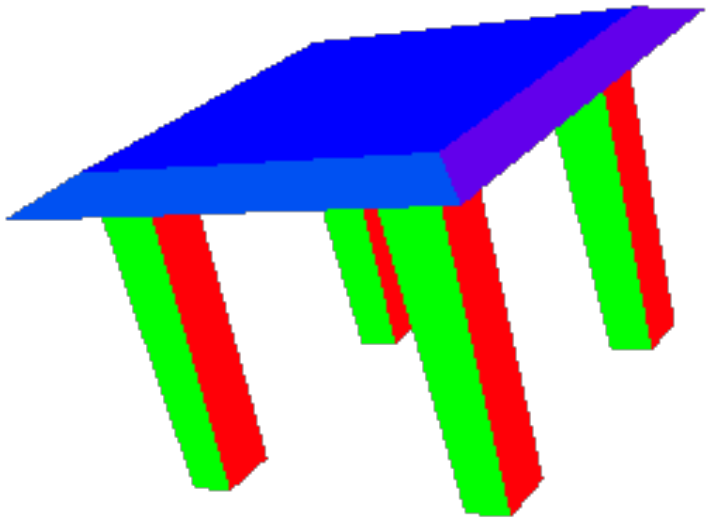
Extended Gaussian Image





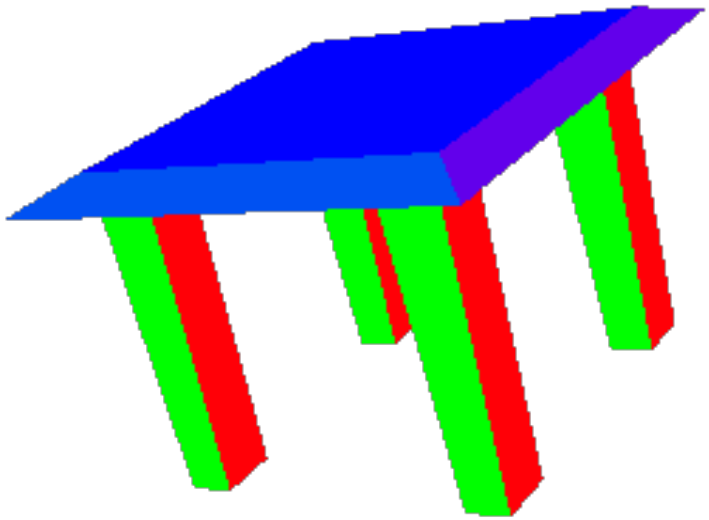
Funkhouser 2004

Extended Gaussian Image



Funkhouser 2004

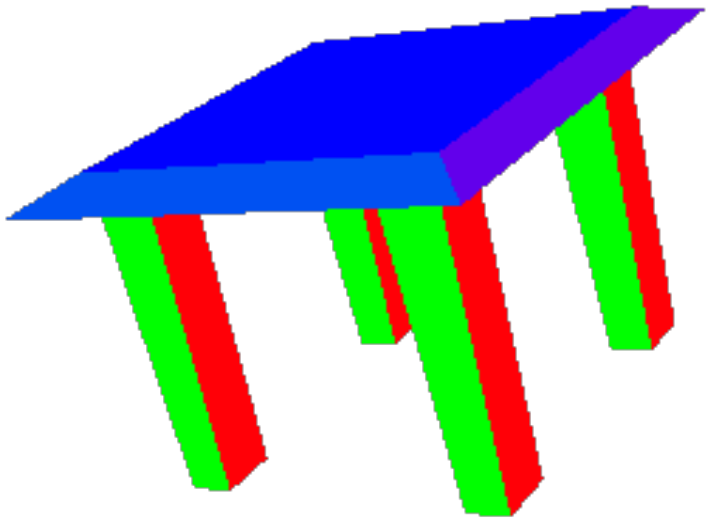
Extended Gaussian Image



- ▷ Efficient To Compute / Concise To Store

Funkhouser 2004

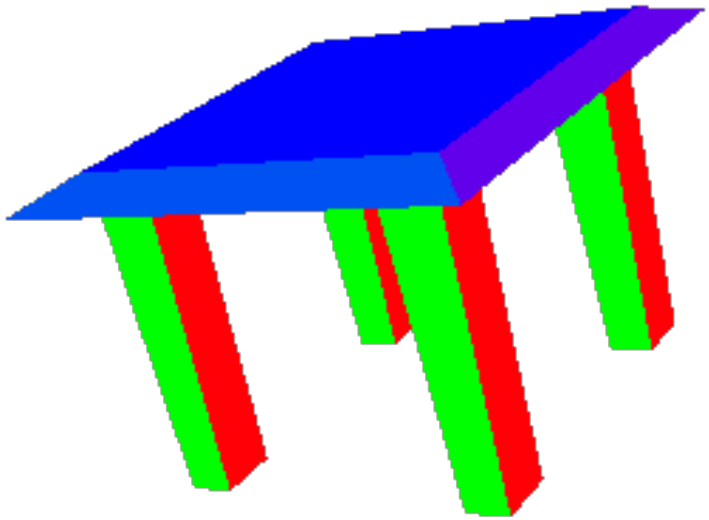
Extended Gaussian Image



- ▷ Efficient To Compute / Concise To Store

Funkhouser 2004

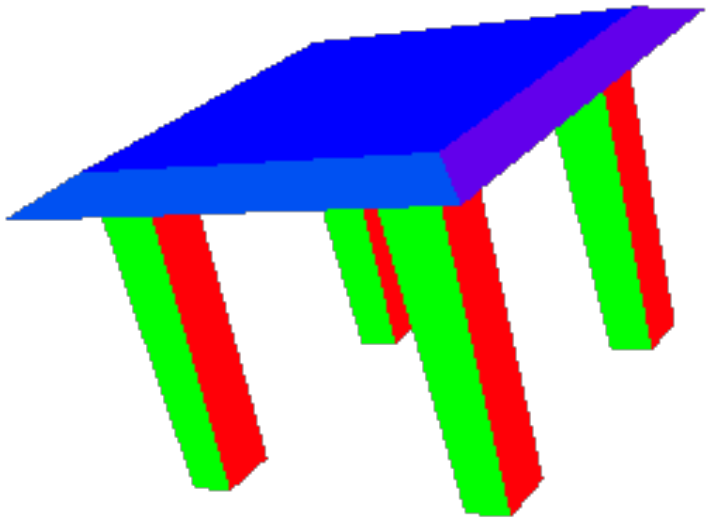
Extended Gaussian Image



- ▷ Discerning

Funkhouser 2004

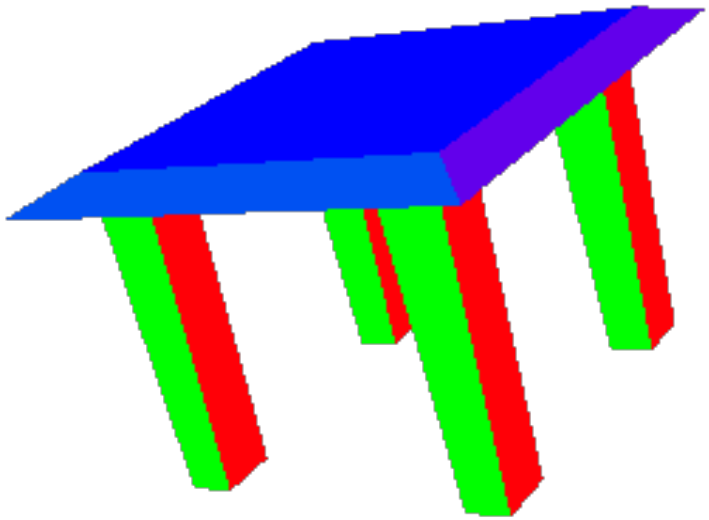
Extended Gaussian Image



- ▷ Discerning (Only fully describes convex objects)

Funkhouser 2004

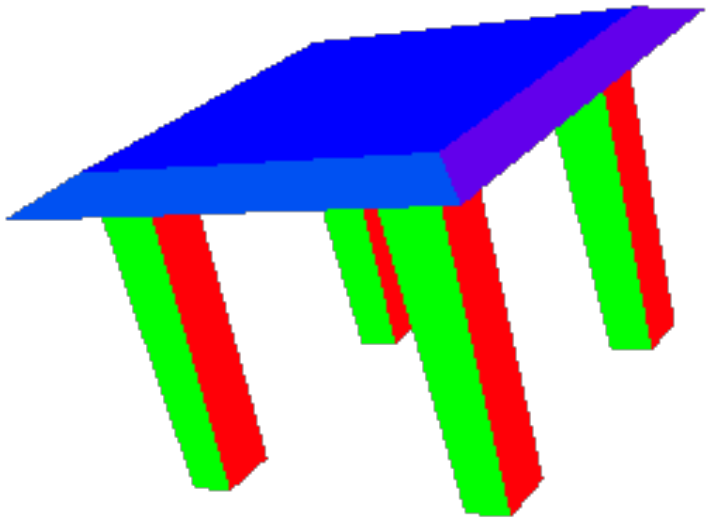
Extended Gaussian Image



- ▷ Rotation Invariant

Funkhouser 2004

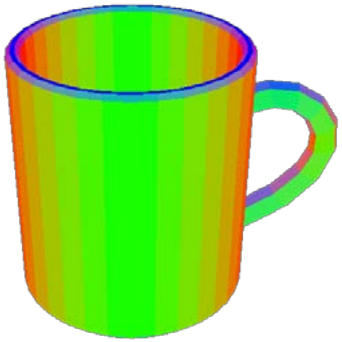
Extended Gaussian Image

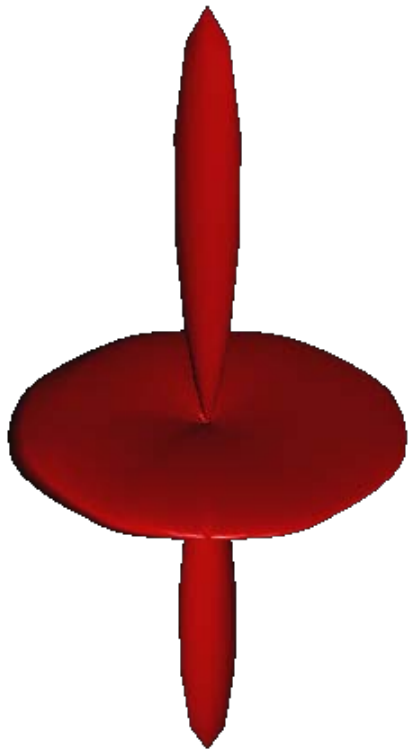


- ▷ Rotation Invariant (Rotate To Align With PCA Axes)

Funkhouser 2004

Extended Gaussian Image

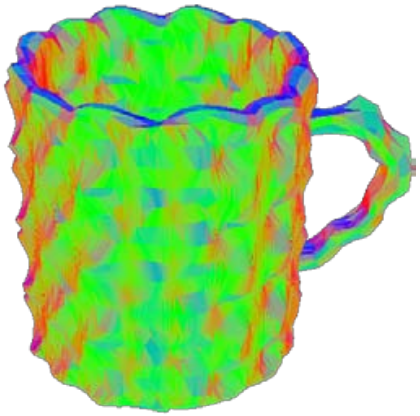




- ▷ Robust To Noise?

Funkhouser 2004

Extended Gaussian Image



- ▷ Not Robust To Noise!

Funkhouser 2004

Table of Contents

Normalize Histograms By Mass

$$h'[i] = \frac{h[i]}{\sum_{k=1}^{N} h[k]}$$

In other words, all bins should sum to 1

Histogram Euclidean Distance

For histograms h_1 and h_2

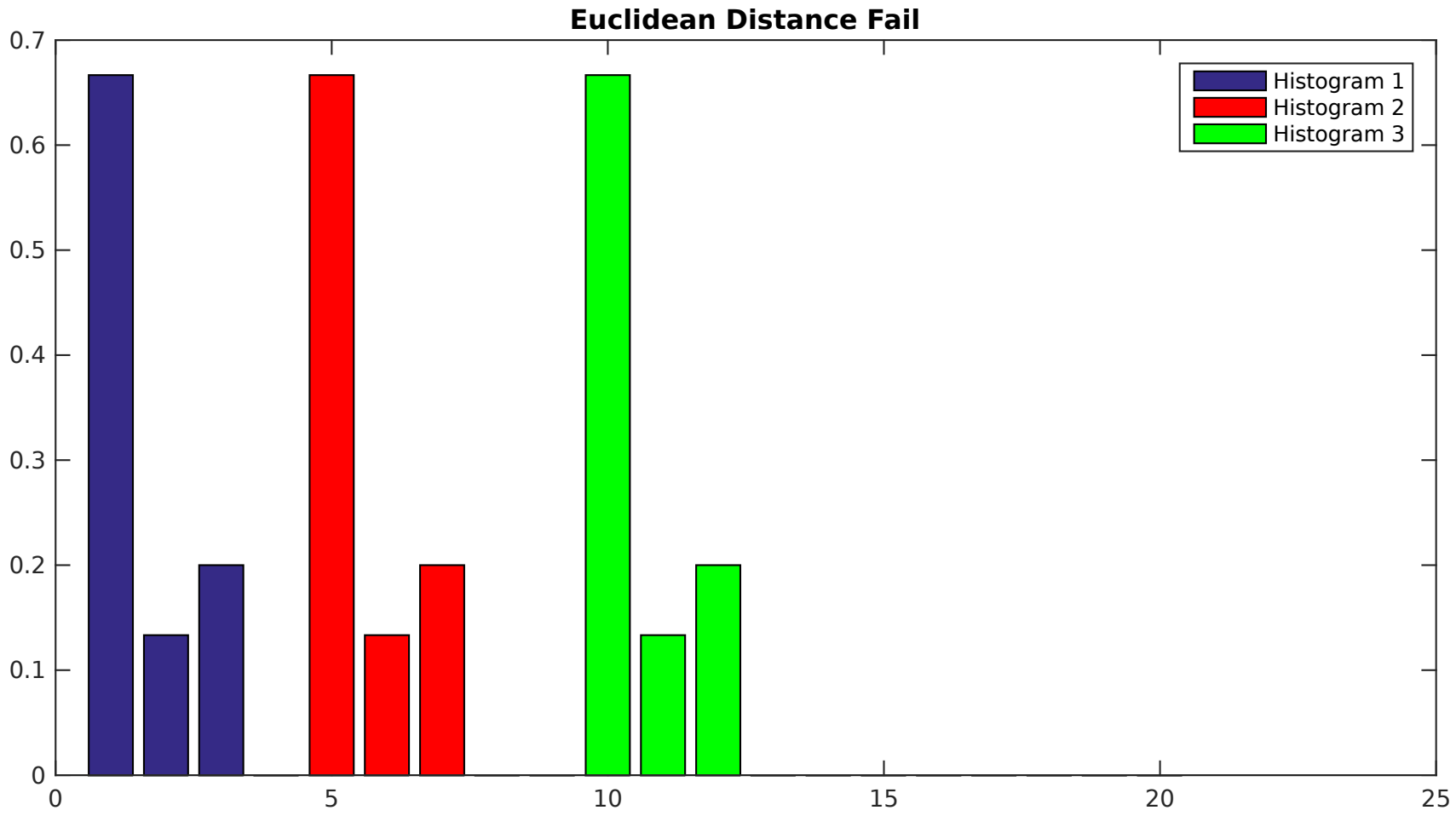
$$d_E(h_1, h_2) = \sqrt{\sum_{i=1}^{N} (h_1[i] - h_2[i])^2}$$

Just thinking of h_1 and h_2 as high dimensional Euclidean vectors! Each histogram bin is a dimension

Histogram Cosine Distance

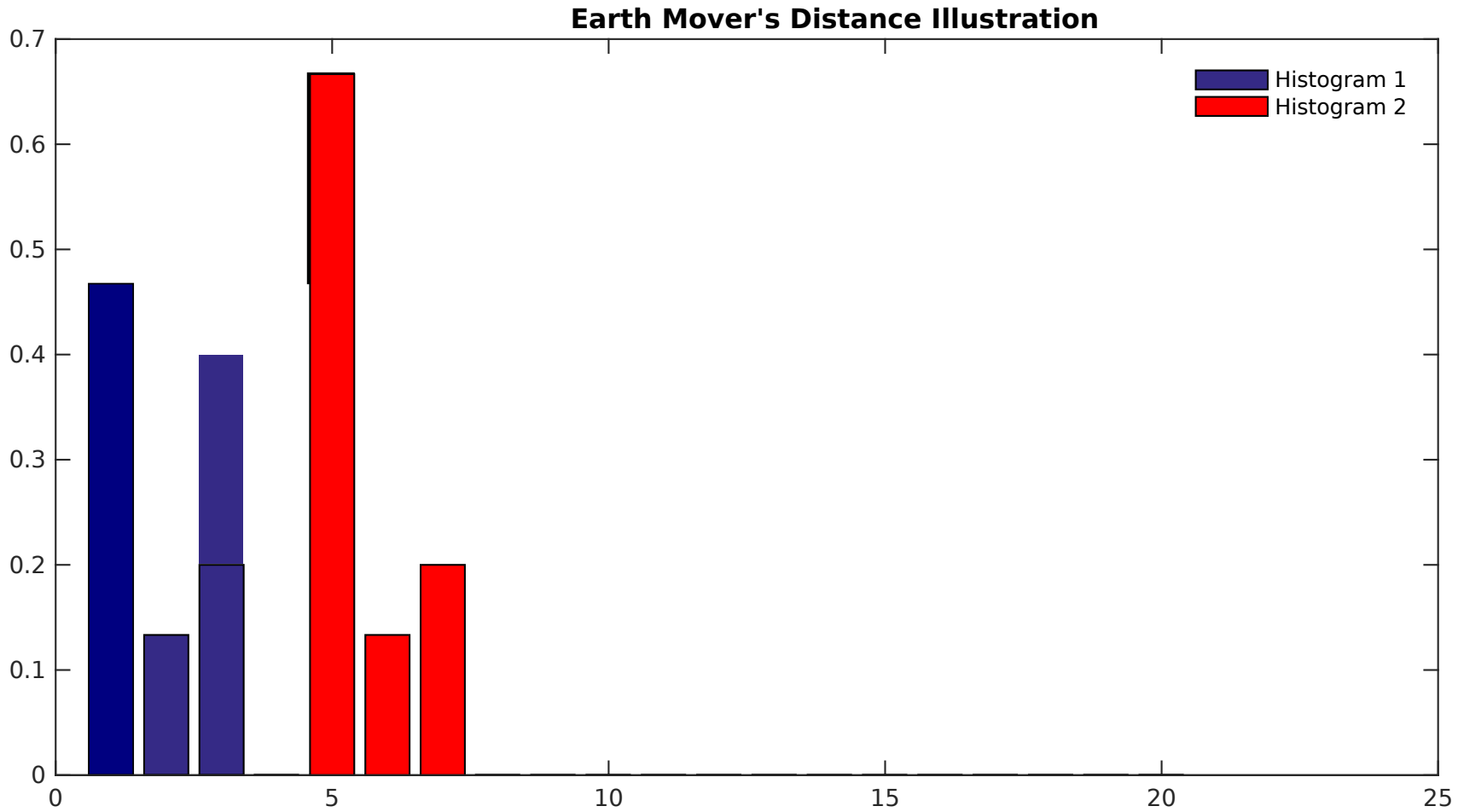
$$d_C(h_1, h_2) = \cos^{-1}\left(\frac{\vec{h_1} \cdot \vec{h_2}}{||\vec{h_1}||||\vec{h_2}||}\right)$$

Euclidean Distance Shortcomings



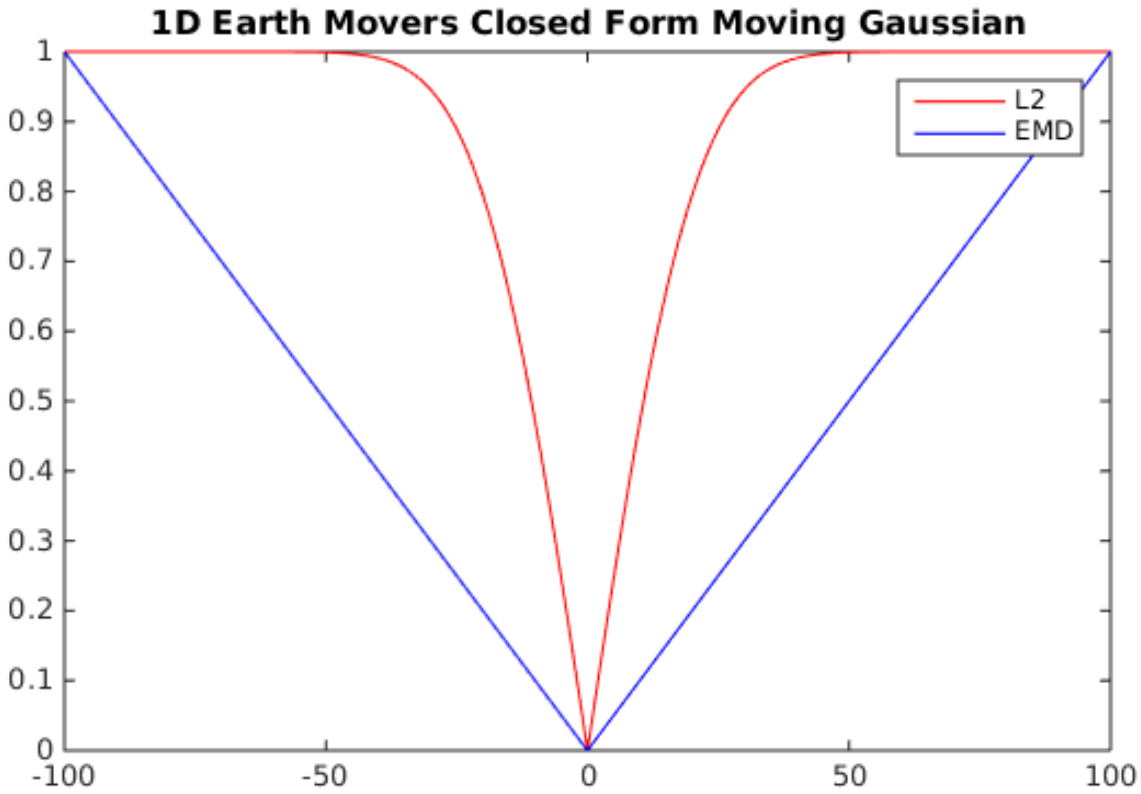
They all have the same distance!

Euclidean Distance Shortcomings



Move earth from blue to red

Earth Mover's Distance



Chi Squared Distance

$$d_\chi(h_1, h_2) = \frac{1}{2} \sum_{i=1}^{N} \frac{(h_1[i] - h_2[i])^2}{h_1[i] + h_2[i]}$$

Exclude values for which $h_1[i] = h_2[i] = 0$

Table of Contents

Evaluation Strategy

Do *leave one out* technique

Use each item as test item in turn, compare to database

- Summarize evaluation statistics over entire database by *averaging them*

Precision / Recall

Rusinkiewiz/Funkhouser 2009

Other Evaluation Metrics

- Average Precision (Area Under Precision/Recall Curve)
- Mean Reciprocal Rank (1/rank of first correct item)
- Median Reciprocal Rank

1 is perfect score