Lecture 15: High Dimensional Data Analysis, Numpy Overview

COMPSCI/MATH 290-04

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3/3/2016

Announcements

- Mini Assignment 3 Out Tomorrow, due next Friday 3/11 11:55PM
- Rank Top 3 Final Project Choices By Tomorrow (Groups of 3-4)
- Dropping Group Assignment 3, Course Grade Schema Change

  Invidiual And Group Programming Assignments 60%
  Final Project 25%
  Midterm Exam 5%
  Class Participation 5%
  Wikipedia Edit 5%
- Midterm Next Thursday 3/10

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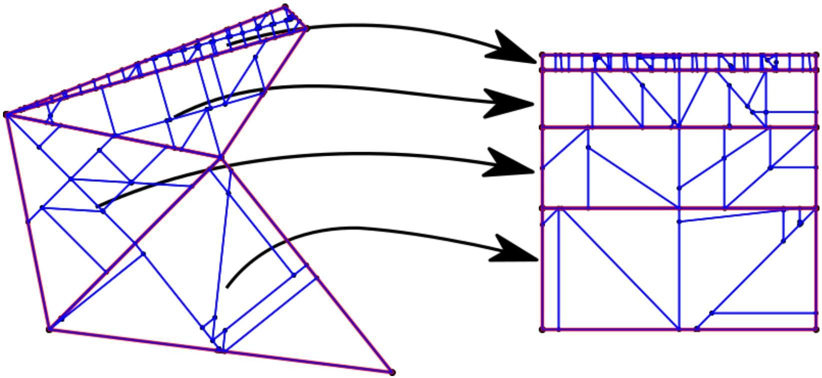
3D Surface Equidecomposability Animation

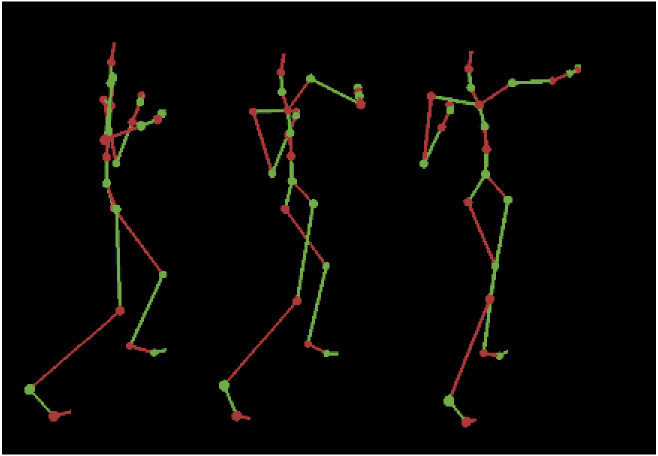


Point Person: Chris Tralie

Ghissi Alterpiece Real Time Rendering

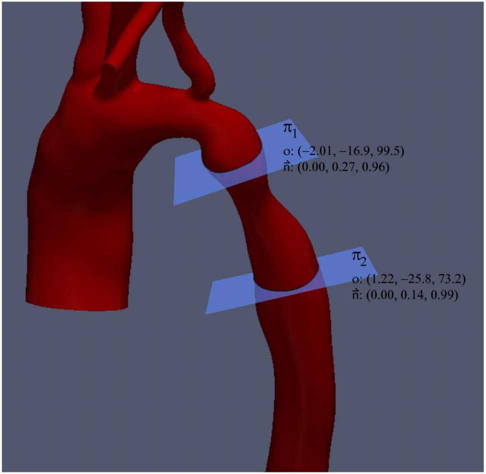
Point Person: Prof Ingrid Daubechies

Motion Capture Javascript Animation



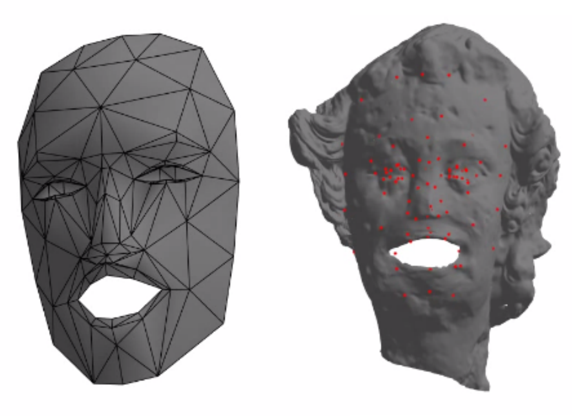
Point People: Chris Tralie / (Prof Ingrid Daubechies?)

Blood Vessel Statistics



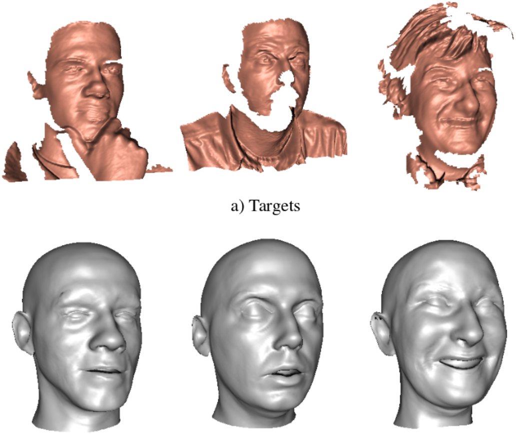
Point People: John Gounley / Prof Amanda Randles

Nasher Museum Talking Heads



Point People: Chris Tralie, Prof Caroline Bruzelius

Face Model Fitting / Morphing



a) Targets

b) Fits

Point People: Jordan Hashemi, Qiang Qiu

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High Dimensional Euclidean Vectors

For *d*-dimensional vectors

$$\vec{a} = (a_1, a_2, \ldots, a_d)$$

$$\vec{b} = (b_1, b_2, \ldots, b_d)$$

High Dimensional Euclidean Vectors

For *d*-dimensional vectors

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$$\vec{b} = (b_1, b_2, \ldots, b_d)$$

Vector addition:

$$a \vec{+} b = (a_1 + b_1, a_2 + b_2, \ldots, a_d + b_d)$$

High Dimensional Euclidean Vectors

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$$\vec{a} = (a_1, a_2, \ldots, a_d)$$

$$\vec{b} = (b_1, b_2, \ldots, b_d)$$

Vector addition:

$$a \vec{+} b = (a_1 + b_1, a_2 + b_2, \ldots, a_d + b_d)$$

Vector subtraction:

$$\vec{ab} = (b_1 - a_1, b_2 - a_2, \ldots, b_d - a_d)$$

High Dimensional Euclidean Vectors

Pythagorean Theorem for

$$\vec{a} = (a_1, a_2, \ldots, a_d)$$

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + \ldots + a_d^2}$$

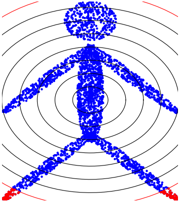
High Dimensional Euclidean Vectors

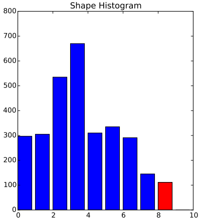
Dot product still holds!

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \ldots + a_d b_d = ||\vec{a}|| ||\vec{b}|| \cos(\theta)$$

Vectors lie on a plane in high dimensions

Histogram Euclidean Distance





For histograms h_1 and h_2

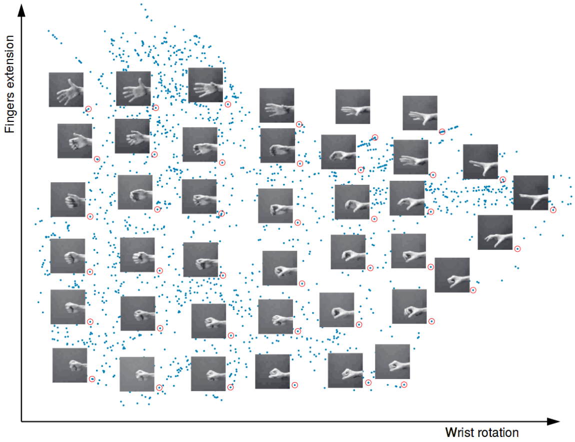
$$d_E(h_1, h_2) = \sqrt{\sum_{i=1}^{N} (h_1[i] - h_2[i])^2}$$

Just thinking of h_1 and h_2 as high dimensional Euclidean vectors! Each histogram bin is a dimension

Histogram Cosine Distance

$$d_C(h_1, h_2) = \cos^{-1}\left(\frac{\vec{h_1} \cdot \vec{h_2}}{||\vec{h_1}||||\vec{h_2}||}\right)$$

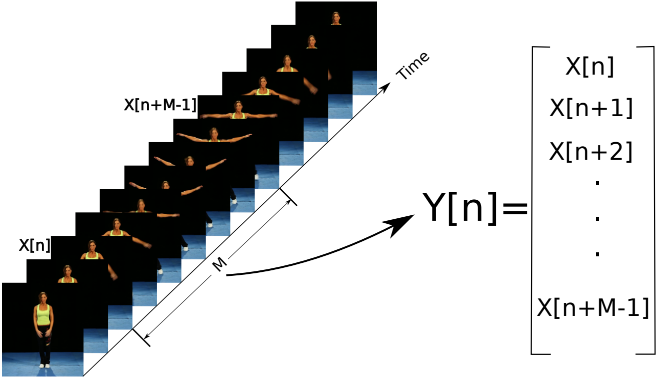
Images Can Be Vectors Too!



One axis per pixel. Above point cloud of images has been flattened to the plane by a nonlinear dimension reduction technique

J. B. Tenenbaum, V. de Silva and J. C. Langford

My Work On Video Loops



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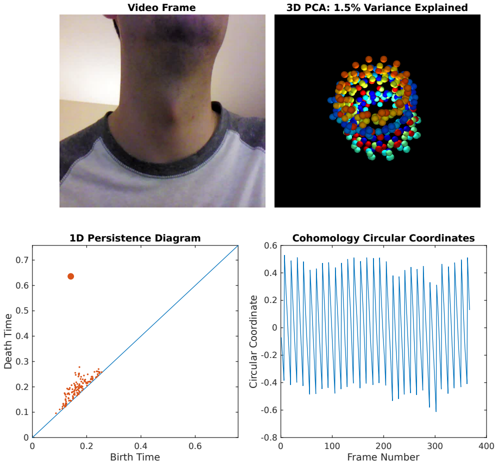
My Work On Video Loops



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Evaluation Strategy

Do *leave one out* technique

Use each item as test item in turn, compare to database

- Summarize evaluation statistics over entire database by *averaging them*

Precision / Recall

Rusinkiewiz/Funkhouser 2009

Other Evaluation Metrics

- Average Precision (Area Under Precision/Recall Curve)
- Mean Reciprocal Rank (1/rank of first correct item)
- Median Reciprocal Rank

1 is perfect score

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Python for This Class

- Use Python 2.7
- Switch your editor to use 4 spaces per tab instead of tabs (!!)
- Required Packages: numpy, matplotlib, pyopengl, wxpython
- Optional Packages: scipy (for some extra tasks)
- Helpful Interactive Code Editing: ipython

Python Basics

```
def doSquare(i):
    return i**2

x = []
for i in range(20):
    if i % 2 == 0:
        continue
    x.append(doSquare(i))

#Do a "list comprehension"
x = [doSquare(val) for val in x]
print x
```

Numpy: Array Basics

Numpy = Python + Matlab

```
import numpy as np

np.random.seed(15) #For repeatable results
X = np.round(5*np.random.randn(4, 3)) #Make a random 4x3
     matrix
print X.shape #Tuple that stores dimensions of array
print X, "\n\n"
#Now do some "array slicing"
print X[:, 0], "\n\n" #Access first column
print X[1, :], "\n\n" #Access, second row
print X[3, 2], "\n\n" #Access fourth row, third column
#Unroll into a 1D array row by row
Y = X.flatten()
print Y.shape
print Y, "\n\n"
Y = Y[:, None]
print Y.shape
print Y
```

Numpy: Randomly Subsample

```python
import numpy as np
import matplotlib.pyplot as plt

#Randomly generate 1000 points
np.random.seed(100) #Seed for repeatable results
NPoints = 1000
X = np.random.randn(2, NPoints)
#Randomly subsample 100 points
NSub = 100
Y = X[:, np.random.permutation(NPoints)[0:NSub]]
plt.plot(X[0, :], X[1, :], '.', color='b')
plt.hold(True) #Don't clear the plot when plotting the
    next thing
plt.scatter(Y[0, :], Y[1, :], 20, color='r')
plt.show()
```

Numpy: Boolean Distance Select

```
import numpy as np
import matplotlib.pyplot as plt

#Randomly generate 1000 points
np.random.seed(100) #Seed for repeatable results
NPoints = 1000
X = np.random.randn(2, NPoints)
#Compute distances of points to origin
R = np.sqrt(np.sum(X**2, 0))
#Select points in X with distance greater than 1
#from origin
Y = X[:, R > 1]
#Plot result
plt.plot(Y[0, :], Y[1, :], '.', color='b')
plt.show()
```

Numpy: Boolean Distance Select

```
import numpy as np
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#Randomly generate 1000 points
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Y = X[:, R > 1]
#Plot result
plt.plot(Y[0, :], Y[1, :], '.', color='b')
plt.show()
```

Numpy: Broadcasting, Rotate Ellipse

```
import numpy as np
import matplotlib.pyplot as plt
np.random.seed(404)
X = np.random.randn(2, 300)
#Scale X by "broadcasting"
X = np.array([[5], [1]])*X
#Setup a rotation matrix
[C, S] = [np.cos(np.pi/4), np.sin(np.pi/4)]
R = np.array([[C, -S], [S, C]])
#Multiply points on the left by the rotation matrix
Y = R.dot(X)
#Set axes equal scale
plt.axes().set_aspect('equal', 'datalim')
plt.plot(Y[0, :], Y[1, :], '.')
plt.show()
```

Numpy: Broadcasting, Sphere Normalization

```
import numpy as np
import matplotlib.pyplot as plt
np.random.seed(404)
X = np.random.randn(2, 300)
#Normalize each column
XNorm = np.sqrt(np.sum(X**2, 0))
#Broadcast 1/XNorm to each row
Y = X/XNorm
plt.plot(Y[0, :], Y[1, :], '.')
plt.show()
```

Numpy: More Broadcasting

```
import numpy as np
import matplotlib.pyplot as plt
X = np.arange(4)
Y = np.arange(6)
Z = X[:, None] + Y[None, :]
print Z
```

Numpy: PCA Implementation

```
import numpy as np
import matplotlib.pyplot as plt
#Make a sinusoid point cloud
t = np.linspace(0, 2*np.pi, 100)
X = np.zeros((2, len(t)))
X[0, :] = t
X[1, :] = np.sin(t)
#Mean-center
X = X-np.mean(X, 1)[:, None]
#Do PCA
D = X.dot(X.T) #X*X Transpose
[eigs, V] = np.linalg.eig(D) #Eigenvectors in columns
eigs = np.sqrt(eigs/X.shape[1]) #Make average dot
    product length
#Scale columns by eigenvectors
V = V*eigs[None, :]
plt.plot(X[0, :], X[1, :], '.'); plt.hold(True)
#First eigvec is in first column, second in second
plt.arrow(0, 0, V[0, 0], V[1, 0], ec = 'r')
plt.arrow(0, 0, V[0, 1], V[1, 1], ec = 'g')
plt.axes().set_aspect('equal', 'datalim'); plt.show()
```

Squared Euclidean Distances in Matrix Form

Notice that

$$||\vec{a}-\vec{b}||^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$||\vec{a}-\vec{b}||^2 = \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{b} - 2\vec{a}\cdot\vec{b}$$

Squared Euclidean Distances in Matrix Form

Notice that

$$||\vec{a} - \vec{b}||^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$||\vec{a} - \vec{b}||^2 = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} - 2\vec{a} \cdot \vec{b}$$

Given points clouds X and Y expressed as $2 \times M$ and $2 \times N$ matrices, respectively, write code to compute an $M \times N$ matrix D so that

$$D[i, j] = ||X[:, i] - Y[:, j]||^2$$

Without using any for loops! Can use for ranking with Euclidean distance or D2 shape histograms, for example

Brute Force Nearest Neighbors

```
import numpy as np
import matplotlib.pyplot as plt
t = np.linspace(0, 2*np.pi, 100)
X = np.zeros((2, len(t)))
X[0, :] = t
X[1, :] = np.cos(t)
Y = np.zeros((2, len(t)))
Y[0, :] = t
Y[1, :] = np.sin(t**1.2)
##FILL THIS IN TO COMPUTE DISTANCE MATRIX D
idx = np.argmin(D, 1) #Find index of closest point in Y
    to point in X
plt.plot(X[0, :], X[1, :], '.')
plt.hold(True)
plt.plot(Y[0, :], Y[1, :], '.', color = 'red')
for i in range(len(idx)):
    plt.plot([X[0, i], Y[0, idx[i]]], [X[1, i], Y[1, idx
        [i]]], 'b')
plt.axes().set_aspect('equal', 'datalim'); plt.show()
```