Lecture 18: Fourier Decomposition, Circular Functions, Spherical Harmonics

COMPSCI/MATH 290-04

Chris Tralie, Duke University

3/22/2016

Announcements

- Midterms graded

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- Group Assignment 1 Graded, Art contest up online (great work!!)

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- Final project directions sent out, first milestone due Friday 4/8
- Ditching Wikipedia entry, final project now worth 30 %

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Why Fourier?? (Interlude)

Hey Chris, isn't this a course on 3D geometry?? Why Fourier???

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Hey Chris, isn't this a course on 3D geometry?? Why Fourier???

- Most CS majors don't know about it, but *extremely important*
- Picks up on "shape" in a different way
- Entry point into harmonic analysis, nonrigid surface statistics

Sinusoid Review

$$f(x) = A\cos(\omega x + \phi)$$

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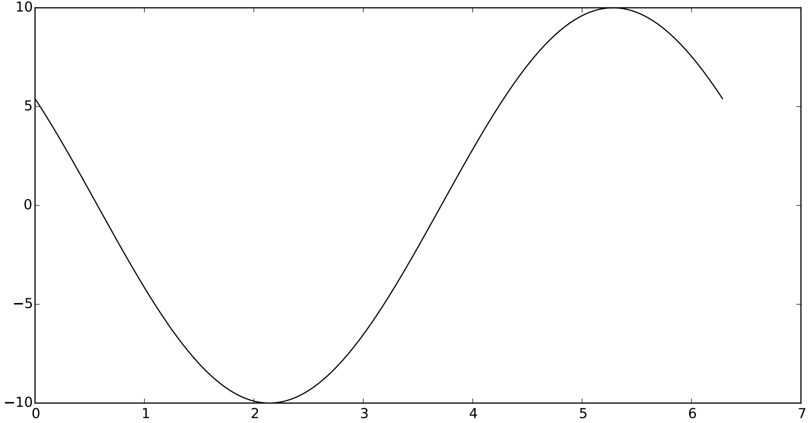
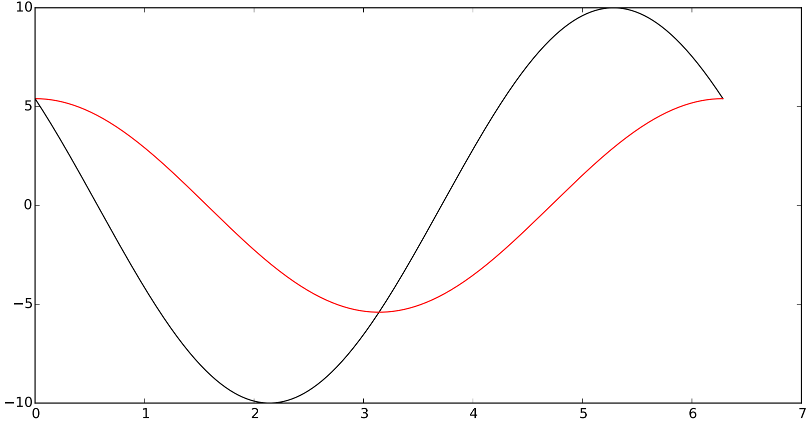
In *polar form*

$$f(x) = Ae^{i(\omega x + \phi)} = Ae^{i\phi}e^{i\omega x} = A(\cos(\theta) + i\sin(\theta))e^{i\omega x}$$

Sinusoid Example



Sinusoid Example



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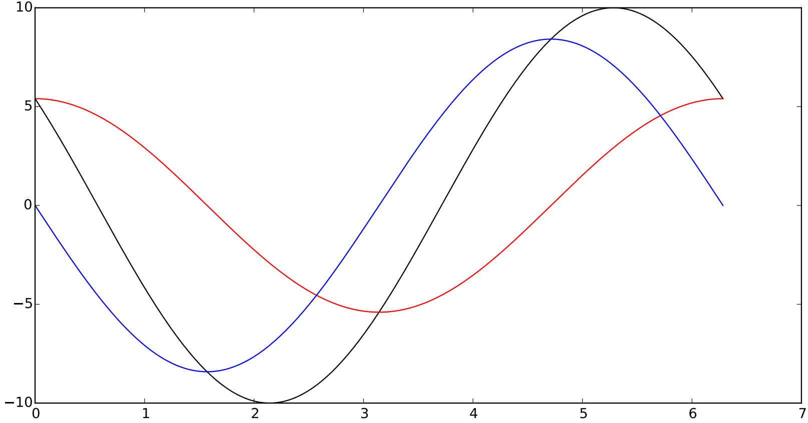
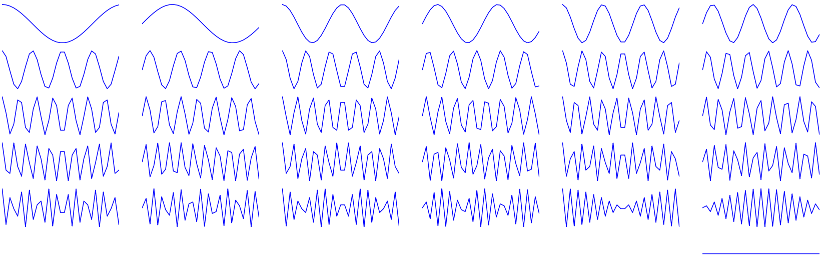


Fourier Decomposition



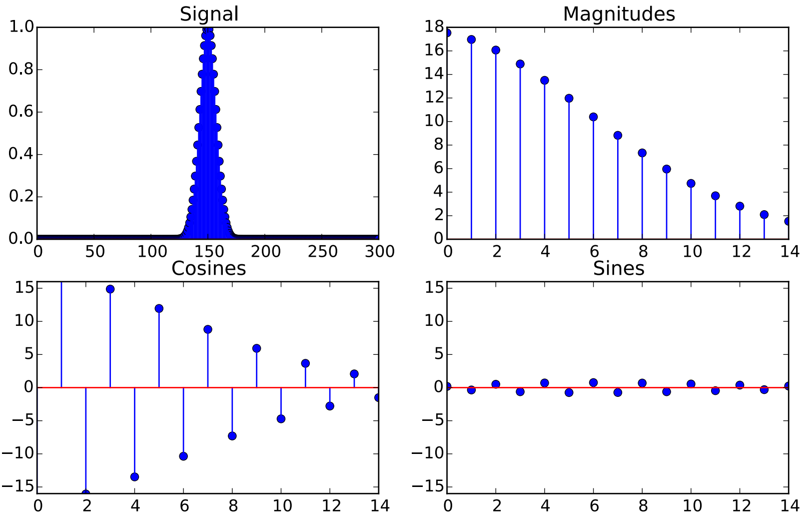
$$f[n] = \sum_{k=0}^{N-1} a_k \cos\left(\frac{2\pi k}{N} n\right) + b_k \sin\left(\frac{2\pi k}{N} n\right)$$

Amplitude at frequency index k is $\sqrt{a_k^2 + b_k^2}$

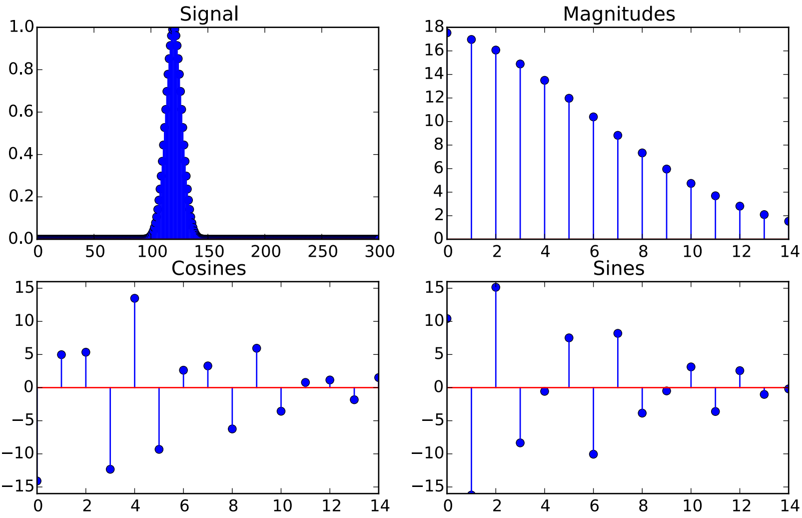
Fourier Decomposition: Gaussian Examples

Show video frames

Fourier Decomposition: Gaussian Examples



Fourier Decomposition: Gaussian Examples



Fourier Decomposition: Ramp Example

Show video frames

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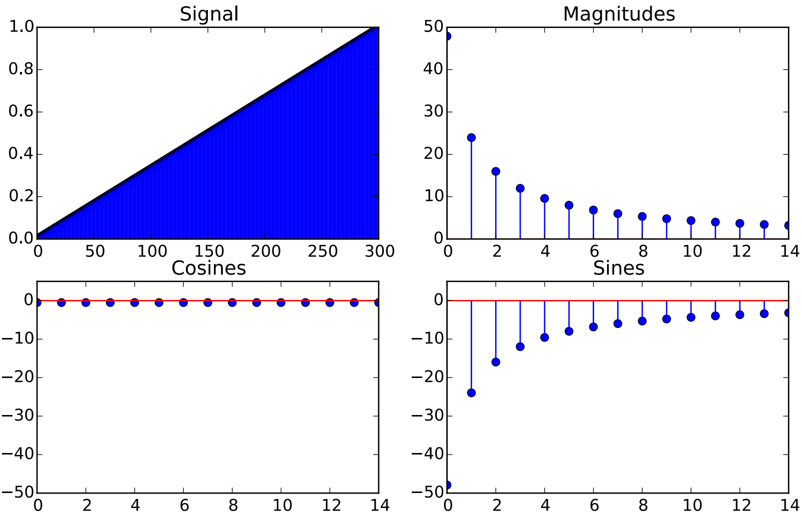

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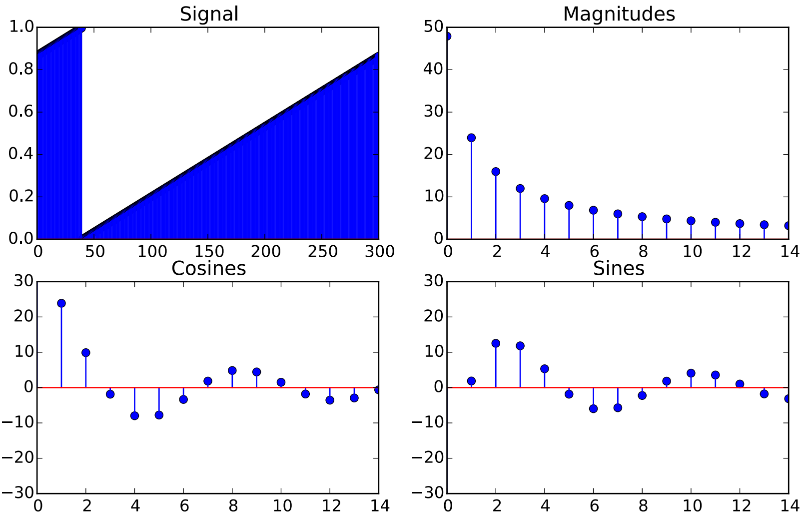

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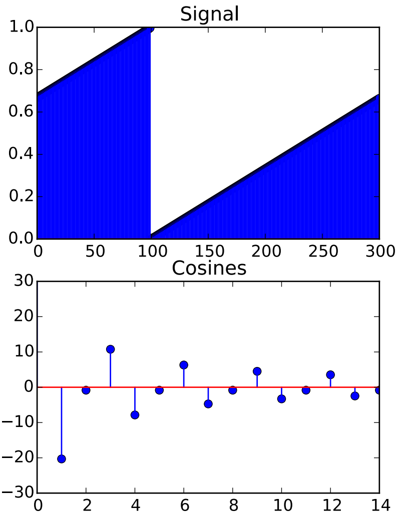
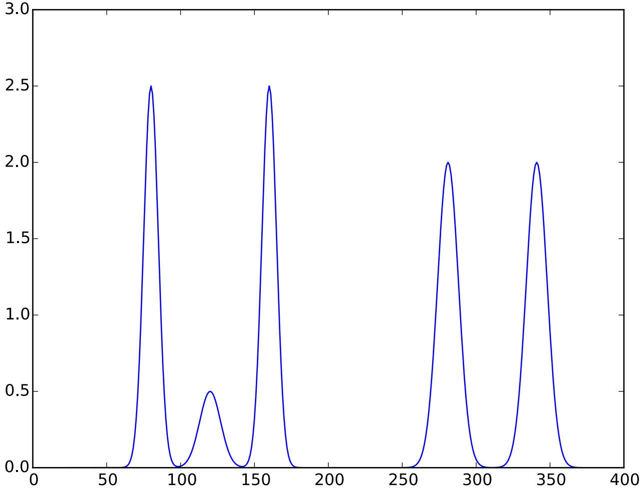


Fourier Decomposition: Ramp Example

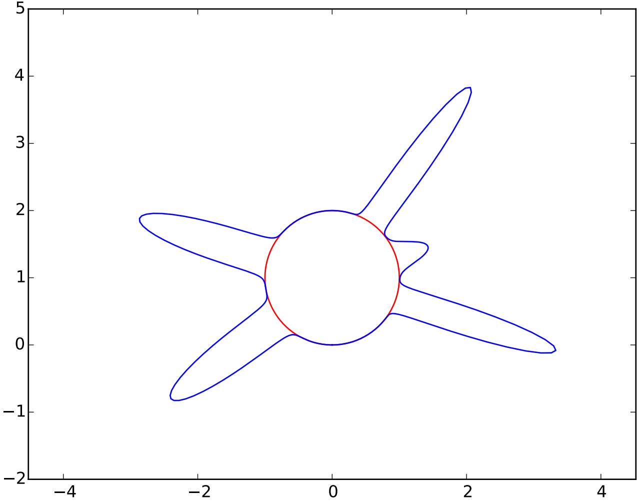
Continuous shifting videos

Functions on The Circle



Show circle wrap video

Functions on The Circle



Phase as a rotation

$$g(x) = f(x + \phi)$$

Show video

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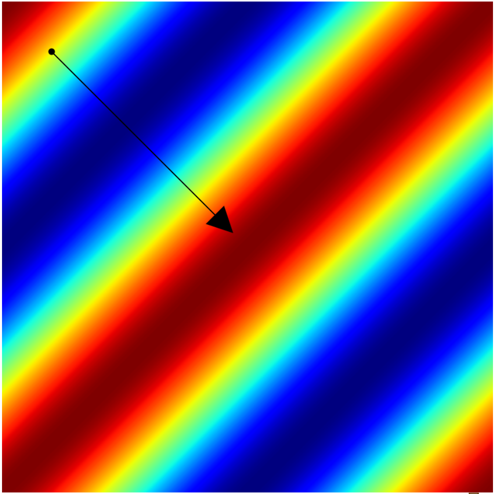
2D Sinusoids (aka "Plane Waves")

$$f(x, y) = \cos(\omega_x x + \omega_y y + \phi)$$

$$f(x, y) = \cos(\vec{\omega} \cdot \vec{x} + \phi)$$

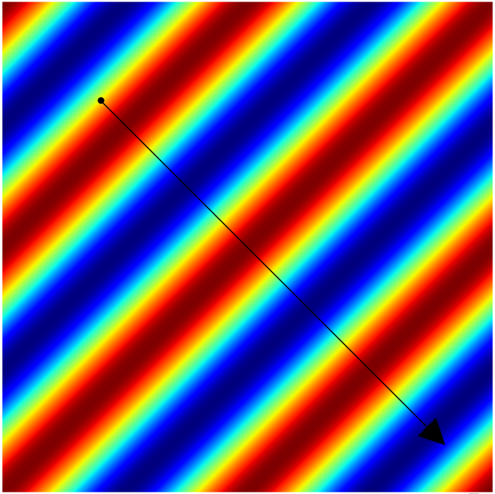
2D Sinusoids: Example

$$f(x,y) = \cos(x+y), \omega_x = 1, \omega_y = 1$$



2D Sinusoids: Example

$$f(x, y) = \cos(2x + 2y), \omega_x = 2, \omega_y = 2$$



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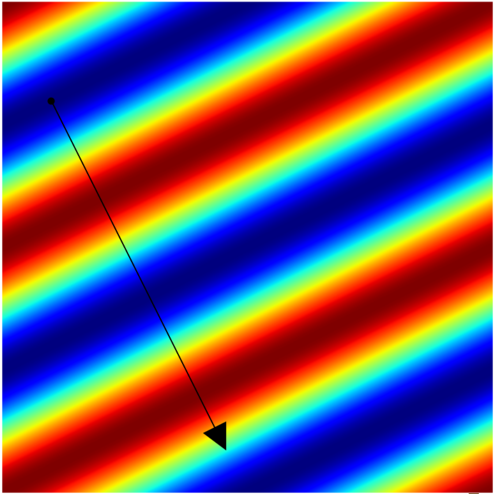
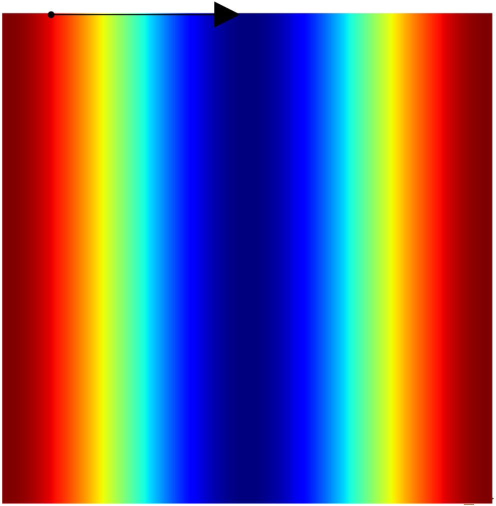


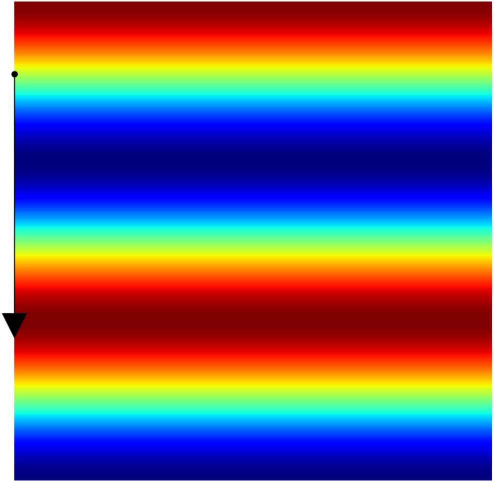
2D Sinusoids: Example

$$f(x, y) = \cos(x), \omega_x = 1, \omega_y = 0$$



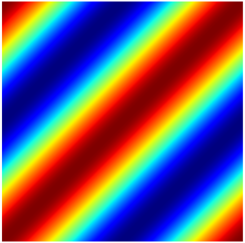
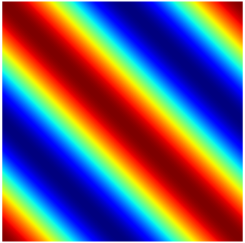
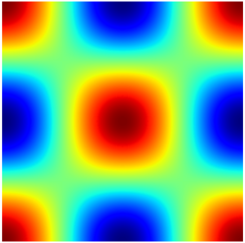
2D Sinusoids: Example

$$f(x, y) = \cos(1.5y), \omega_x = 0, \omega_y = 1.5$$



2D Sinusoids: Interference Pattern

$$f(x, y) = \cos(x + y) + \cos(x - y)$$

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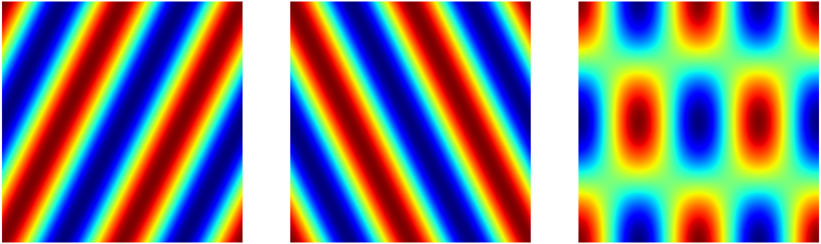
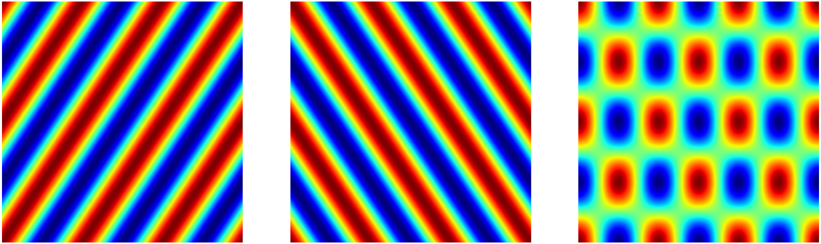
$$f(x, y) = \cos(2x + y) + \cos(2x - y)$$



2D Sinusoids: Interference Pattern

$$f(x,y) = \cos(3x + 2y) + \cos(3x - 2y)$$



2D Sinusoids: Interference

Why is this happening?

$$g(x, y) = \cos(\omega_x x + \omega_y y) + \cos(\omega_x x - \omega_y y)$$

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$$g(x,y) = 2\cos(\omega_x x)\cos(\omega_y y)$$

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Spherical Coordinates Review

Spherical Harmonics

$$Y_l^m \propto P_l^m(\cos(\theta))e^{im\phi}$$

Interactive Demo

Spherical Harmonic Shape Descriptors

Show Tom's paper