Lecture 19: Introduction To Topology

COMPSCI/MATH 290-04

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3/24/2016

Announcements

- Group Assignment 2 Due Wednesday 3/30
- First project milestone Friday 4/8/2016
- Welcome to unit 3!

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Graphs Review

Planar Graphs

The Euler Characteristic

$$\chi = V - E + F$$

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Planar graphs?

The Euler Characteristic

$$\chi = V - E + F = 2$$

Planar graphs?

The Euler Characteristic: Proof

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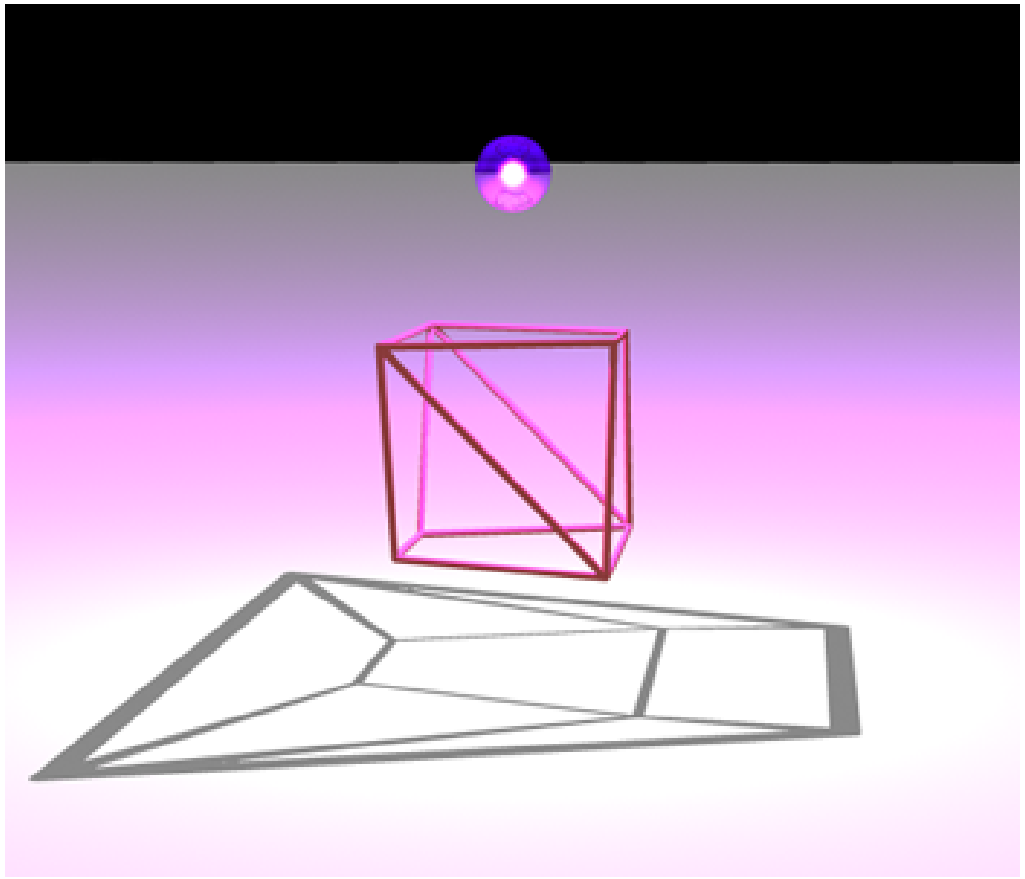
Regular Polygons

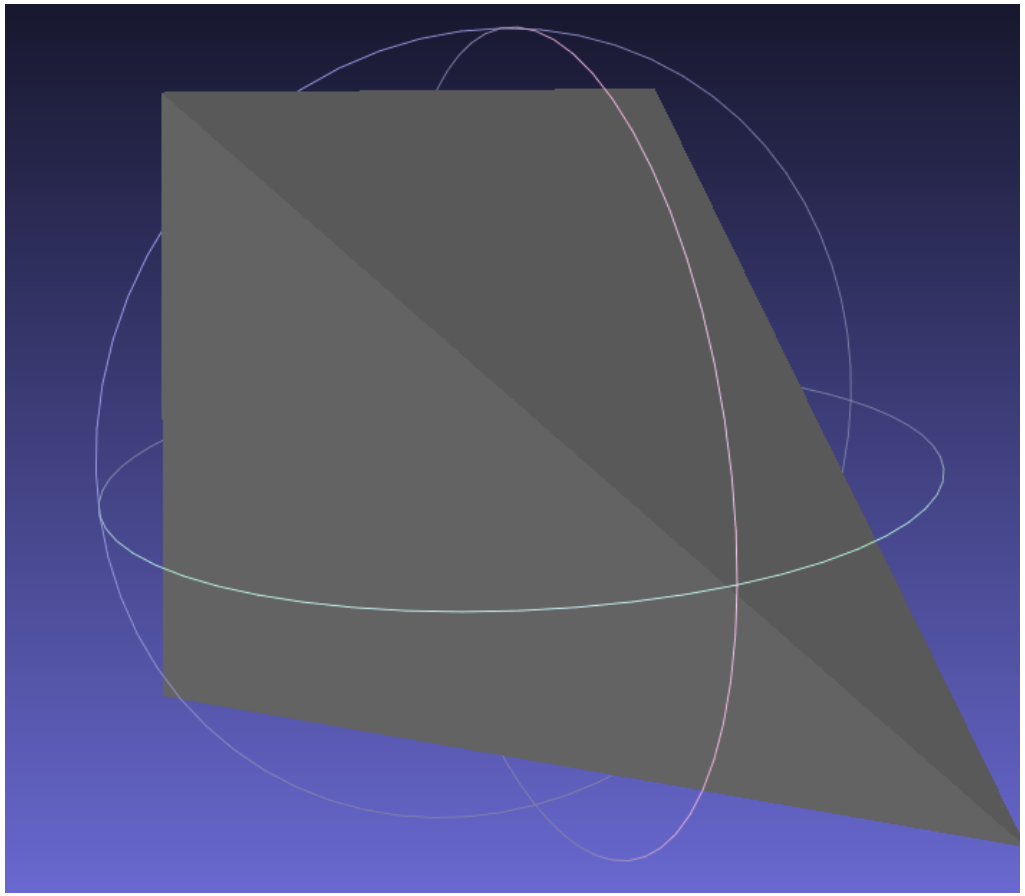
Stereographic Projection



http://www.ics.uci.edu/~eppstein/junkyard/euler/

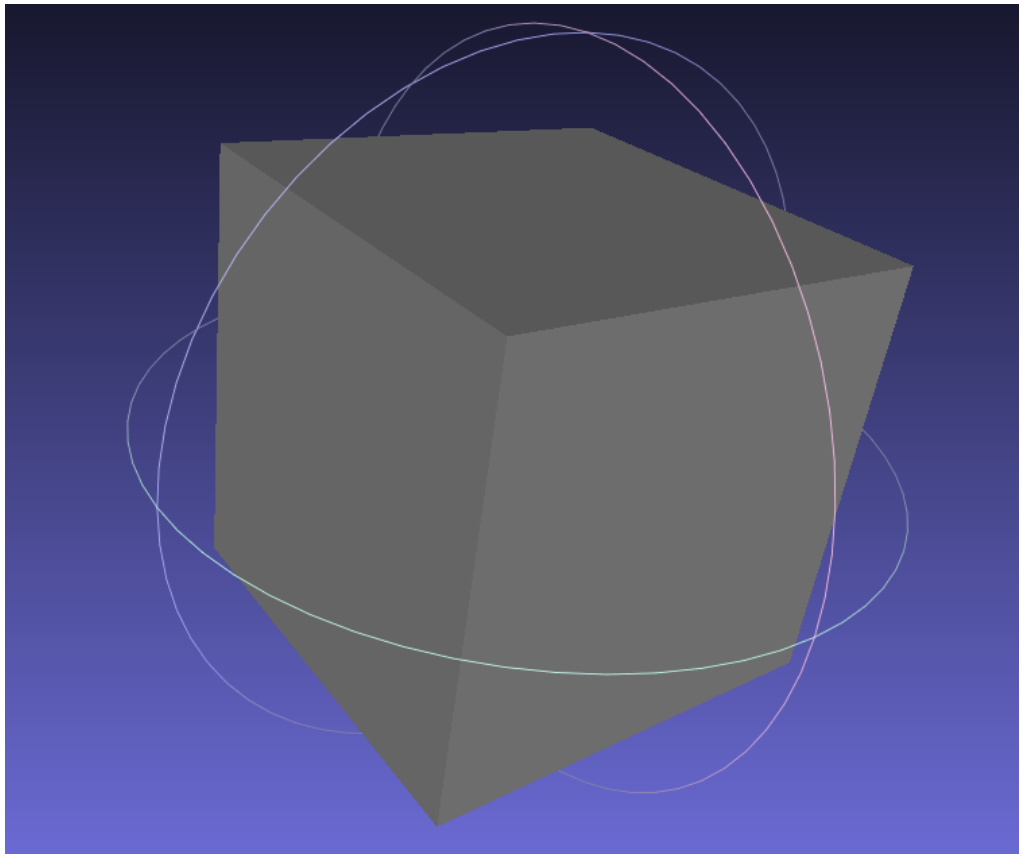
Regular Polyhedra (Platonic Solids)

The Tetrahedron: 4 Vertices, 4 Faces, Triangle Faces



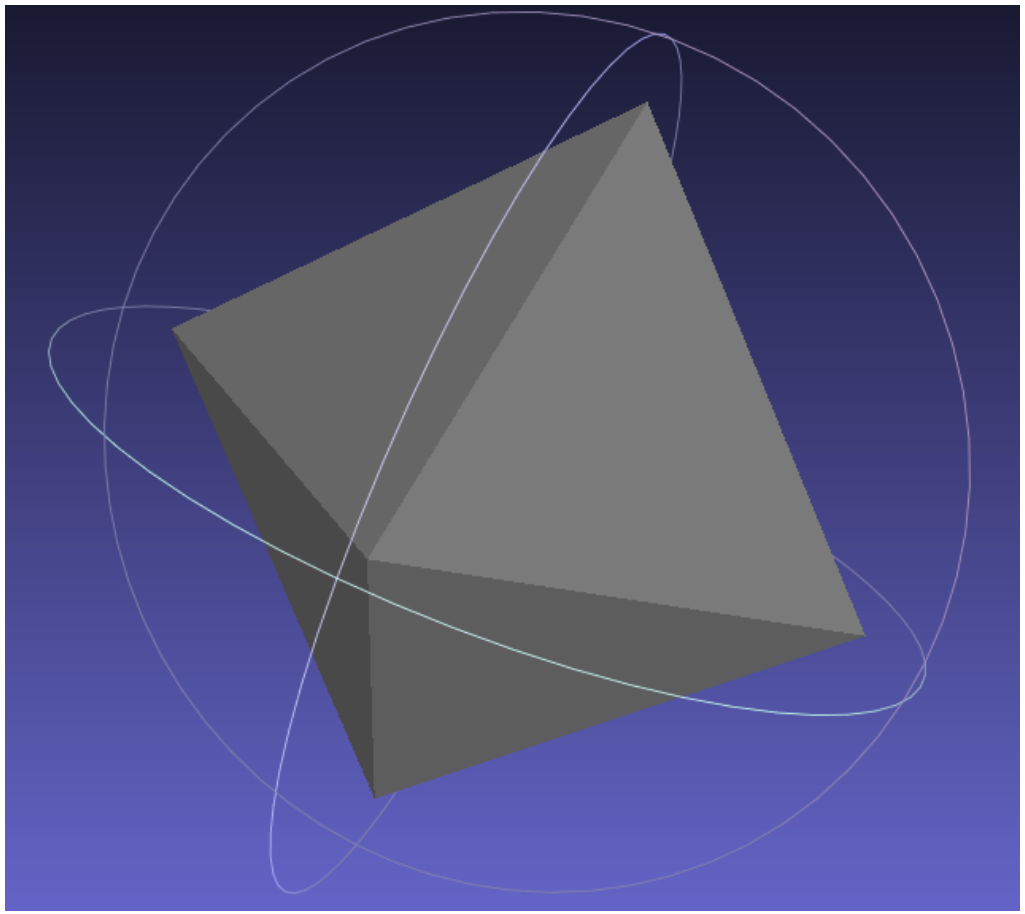
Regular Polyhedra (Platonic Solids)

The Cube: 8 Vertices, 6 Faces, Square Faces



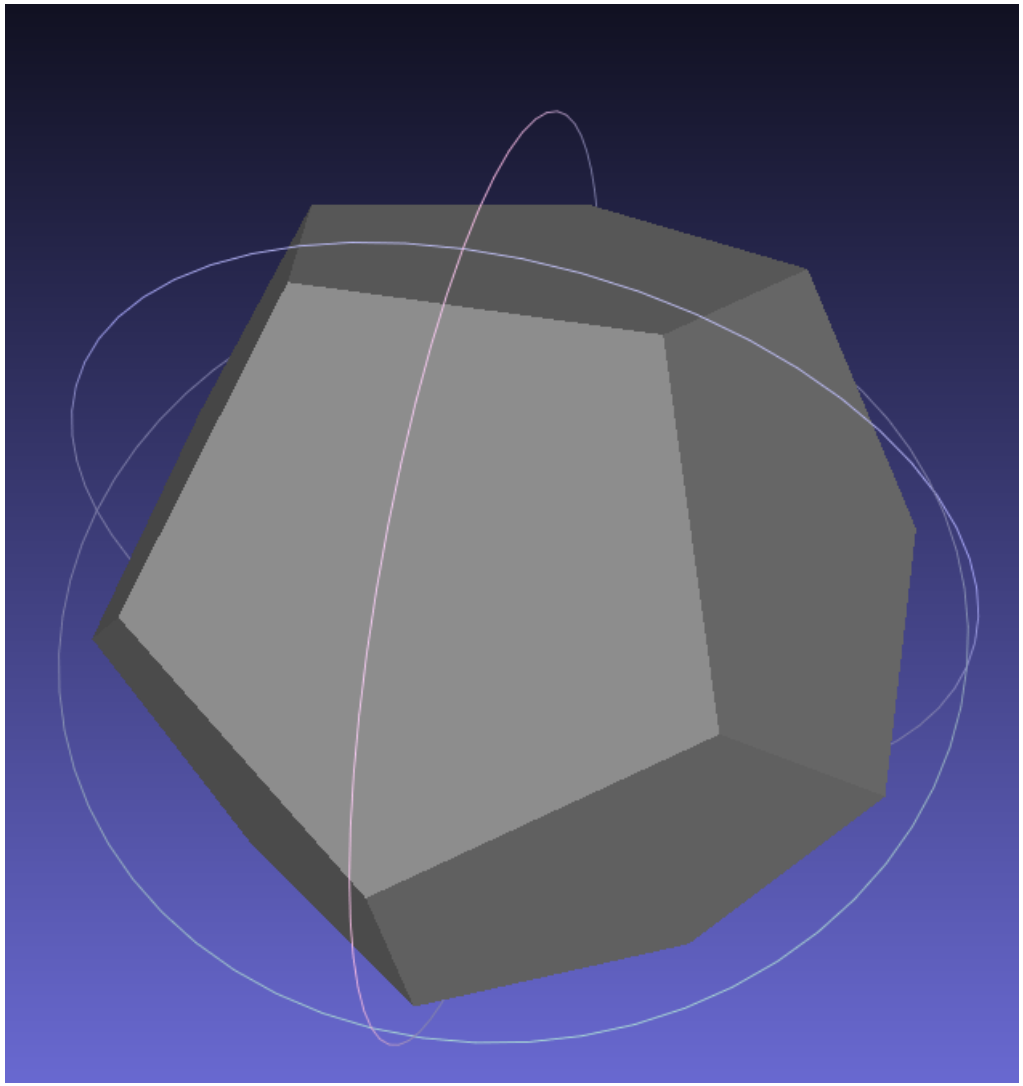
Regular Polyhedra (Platonic Solids)

The Octahedron: 6 Vertices, 8 Faces, Triangle Faces



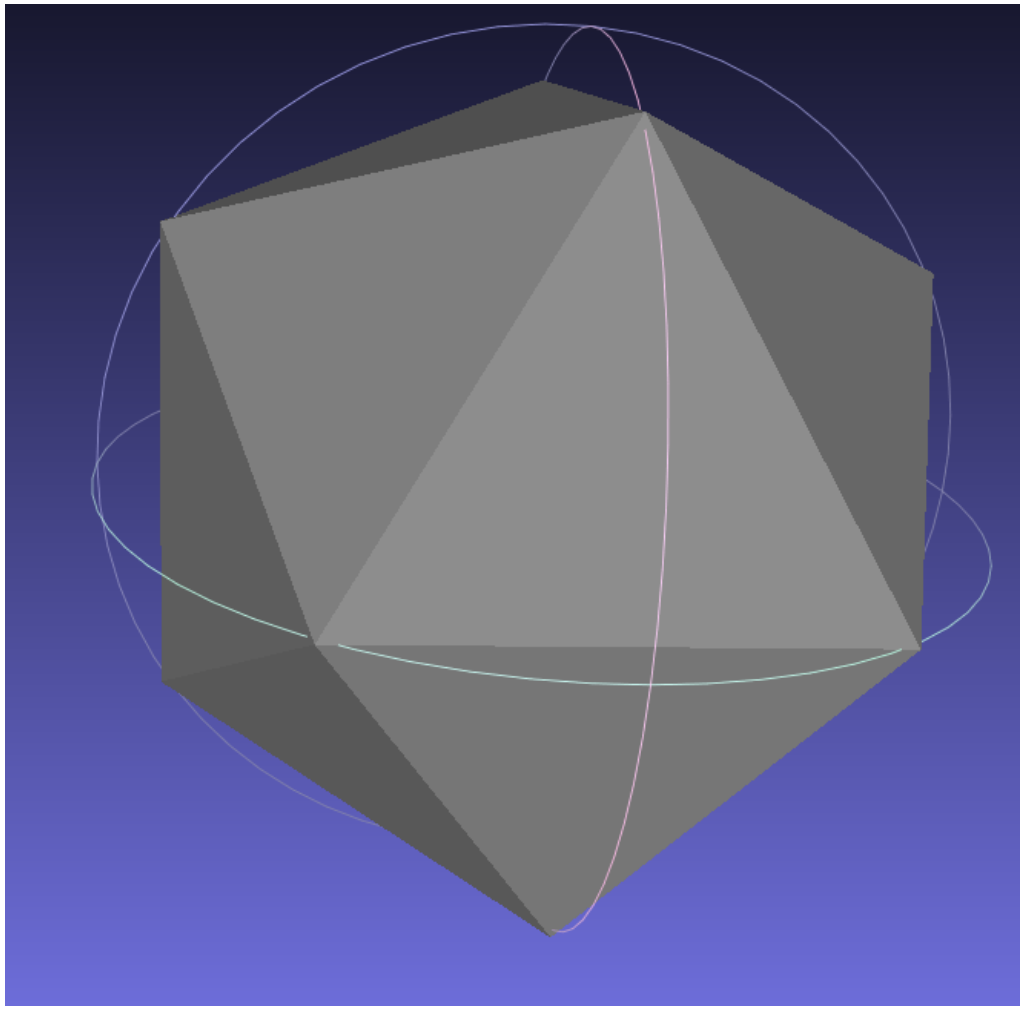
Regular Polyhedra (Platonic Solids)

The Dodecahedron: 20 Vertices, 12 Faces, Pentagonal Faces

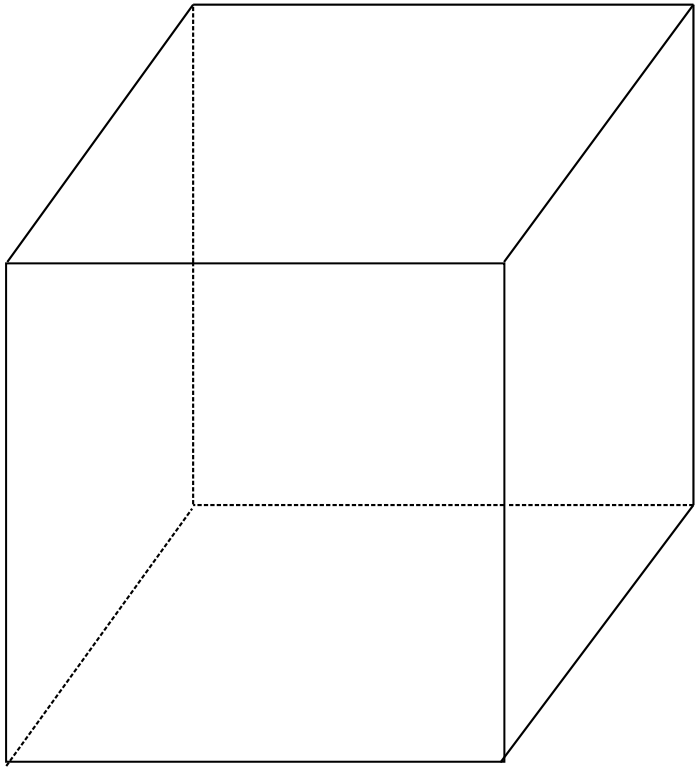


Regular Polyhedra (Platonic Solids)

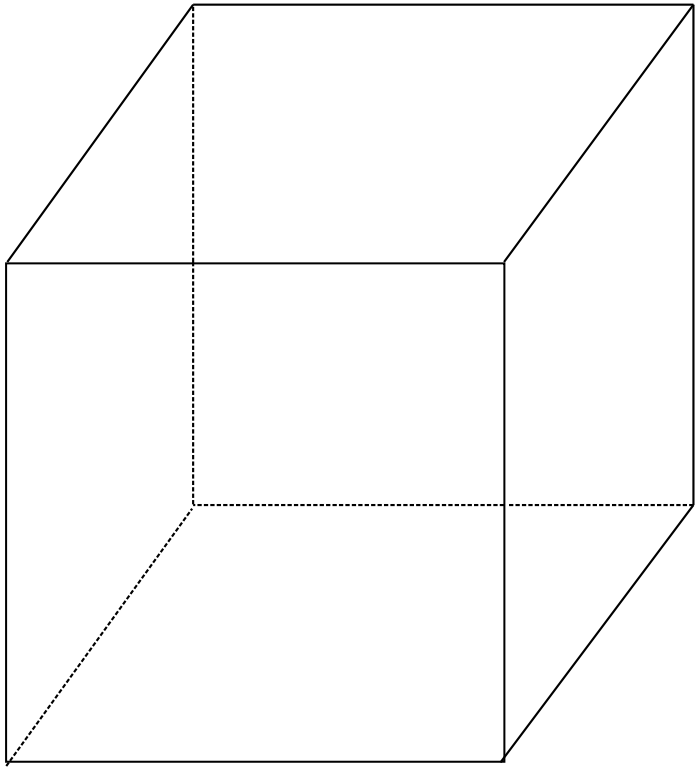
The Icosahedron: 12 Vertices, 20 Faces, Triangle Faces



Constructing The Tetrahedron



Constructing The Icosahedron



Platonic Solids: Is This it??

Let p be the number of sides per face, q be the *degree* of each vertex

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Combine with $V - E + F = 2$

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$$\frac{1}{q} + \frac{1}{p} = \frac{1}{2} + \frac{1}{E}$$

$$\Longrightarrow \frac{1}{q} + \frac{1}{p} > \frac{1}{2}$$

Flattening To Plane

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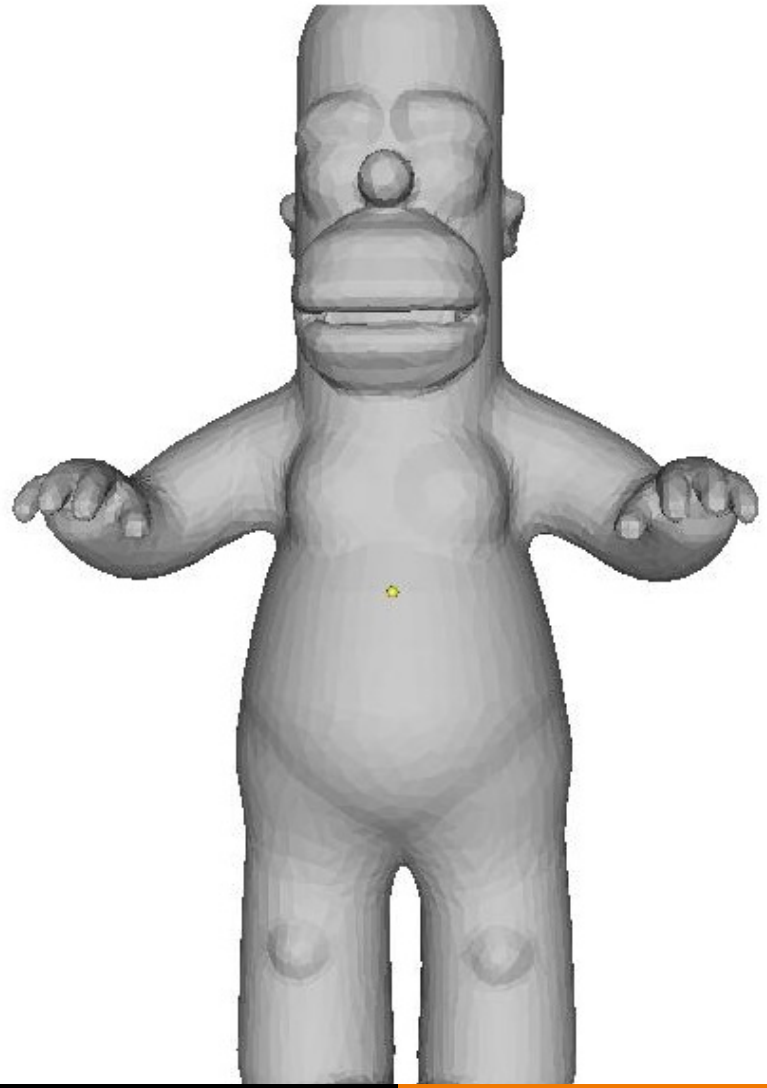
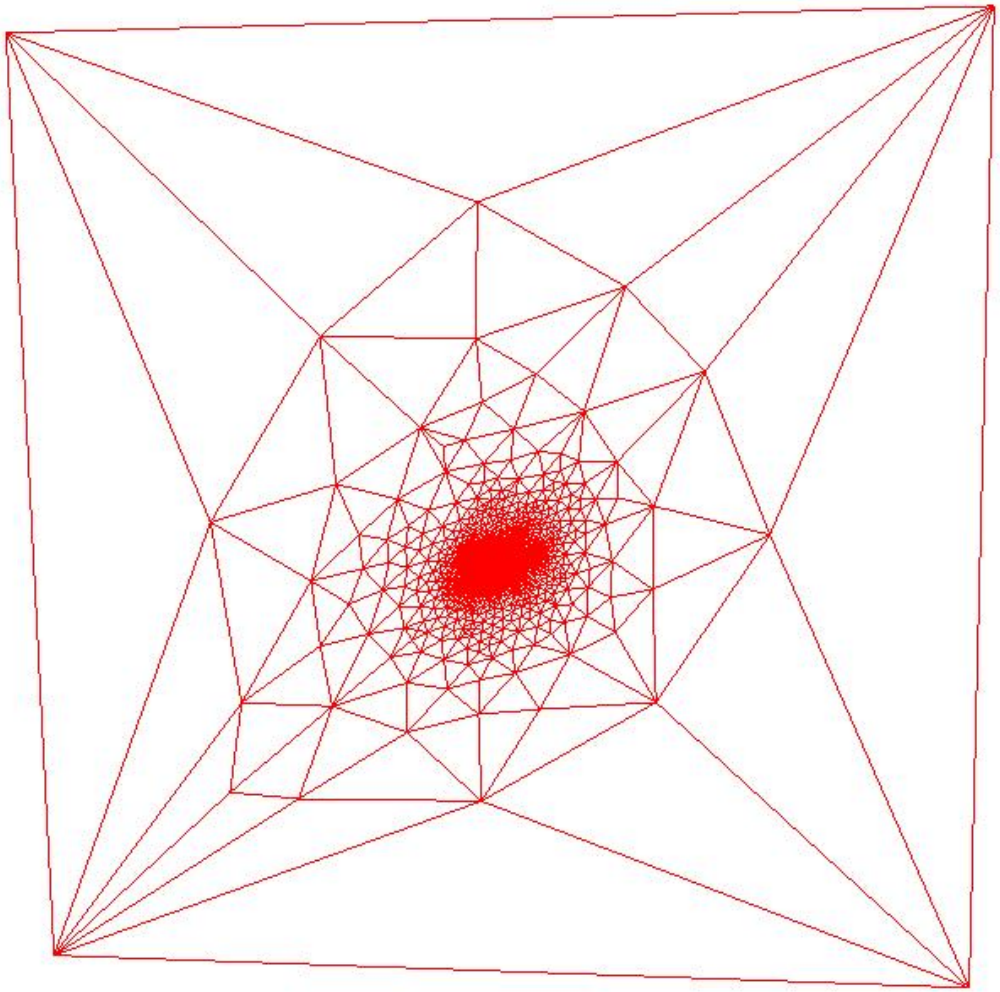


Flattening To Plane



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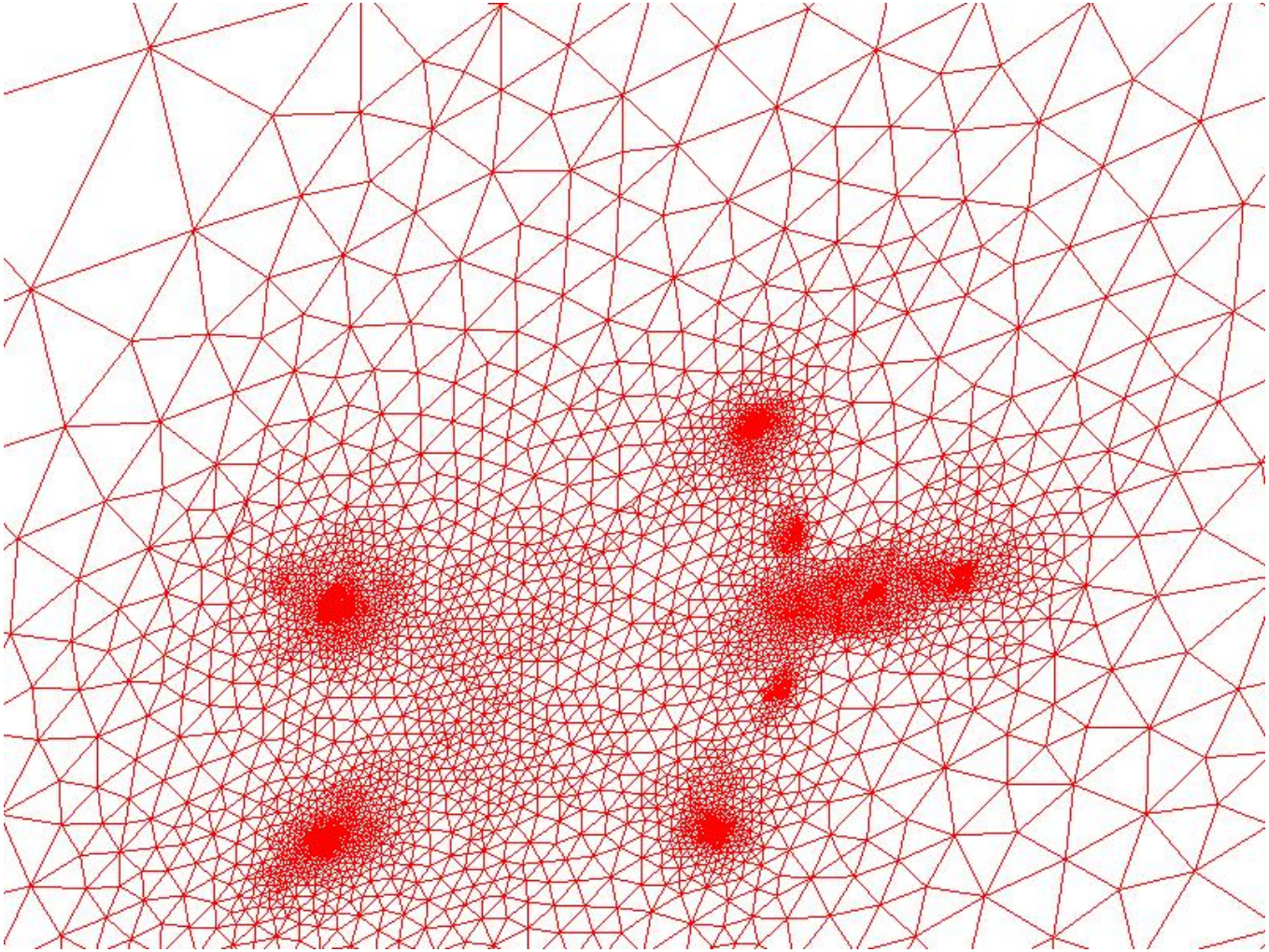
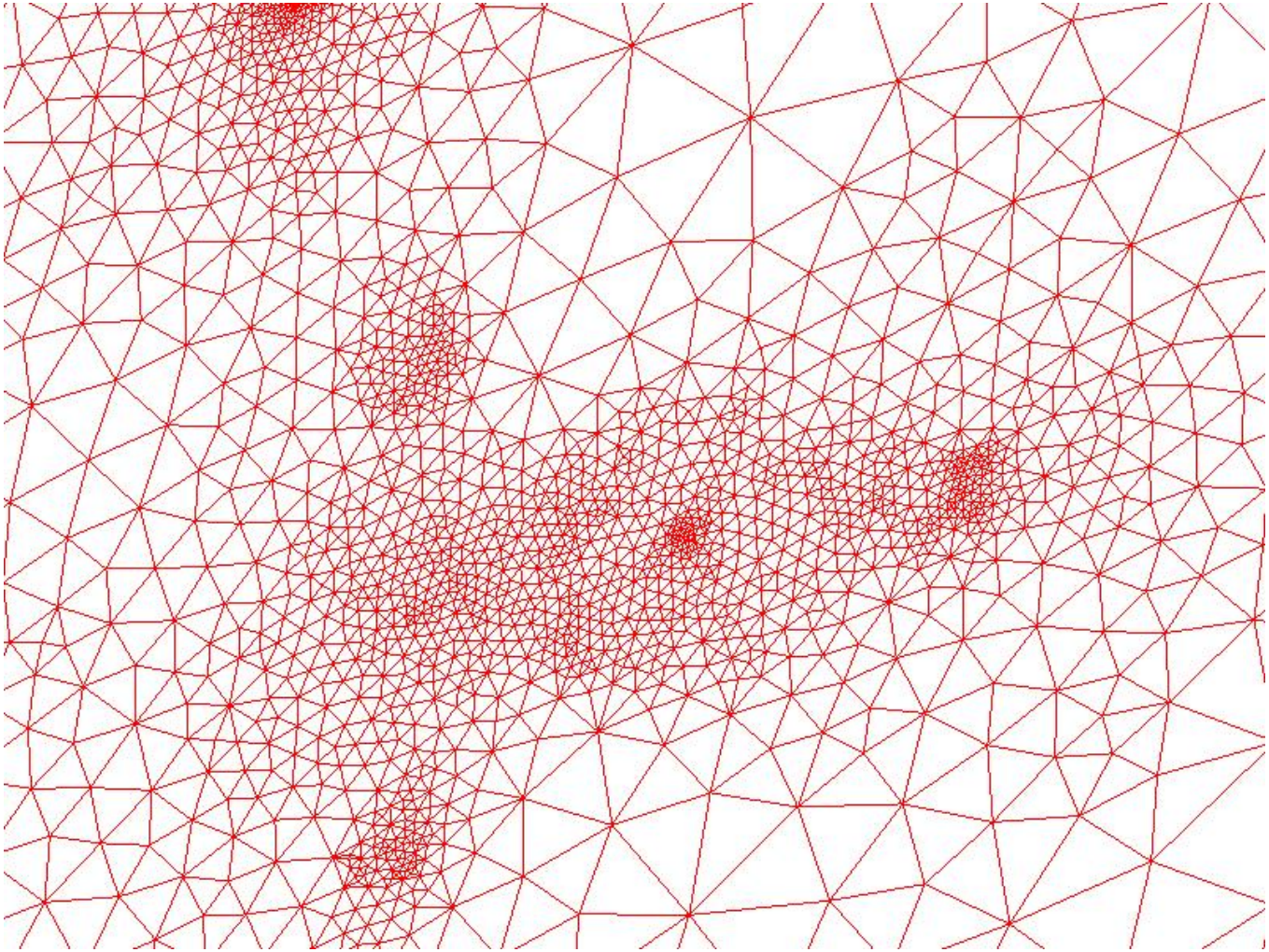


Flattening To Plane



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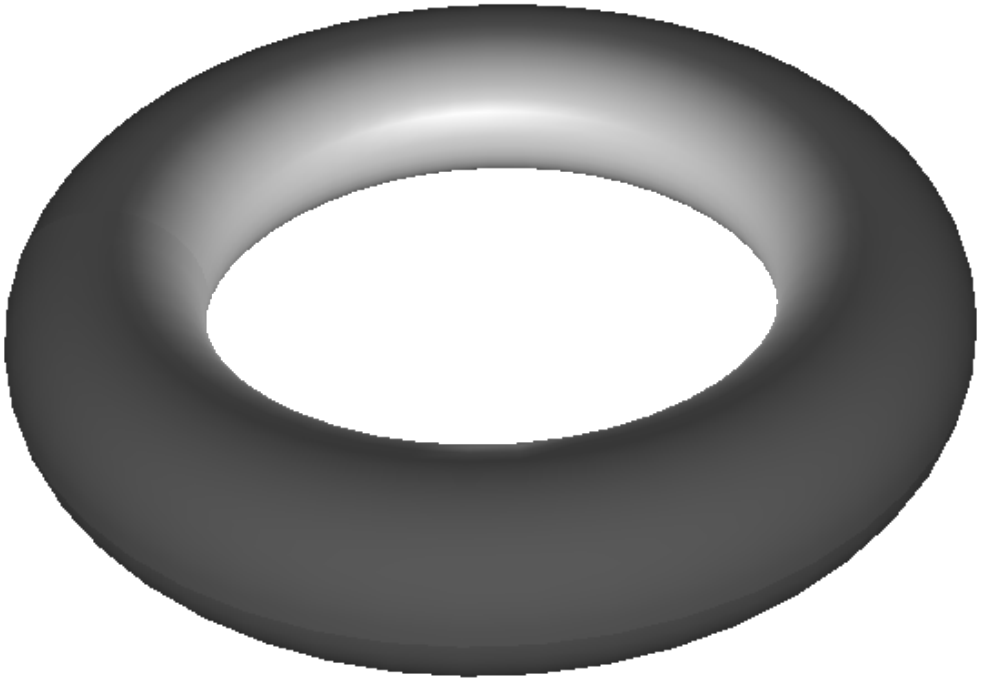
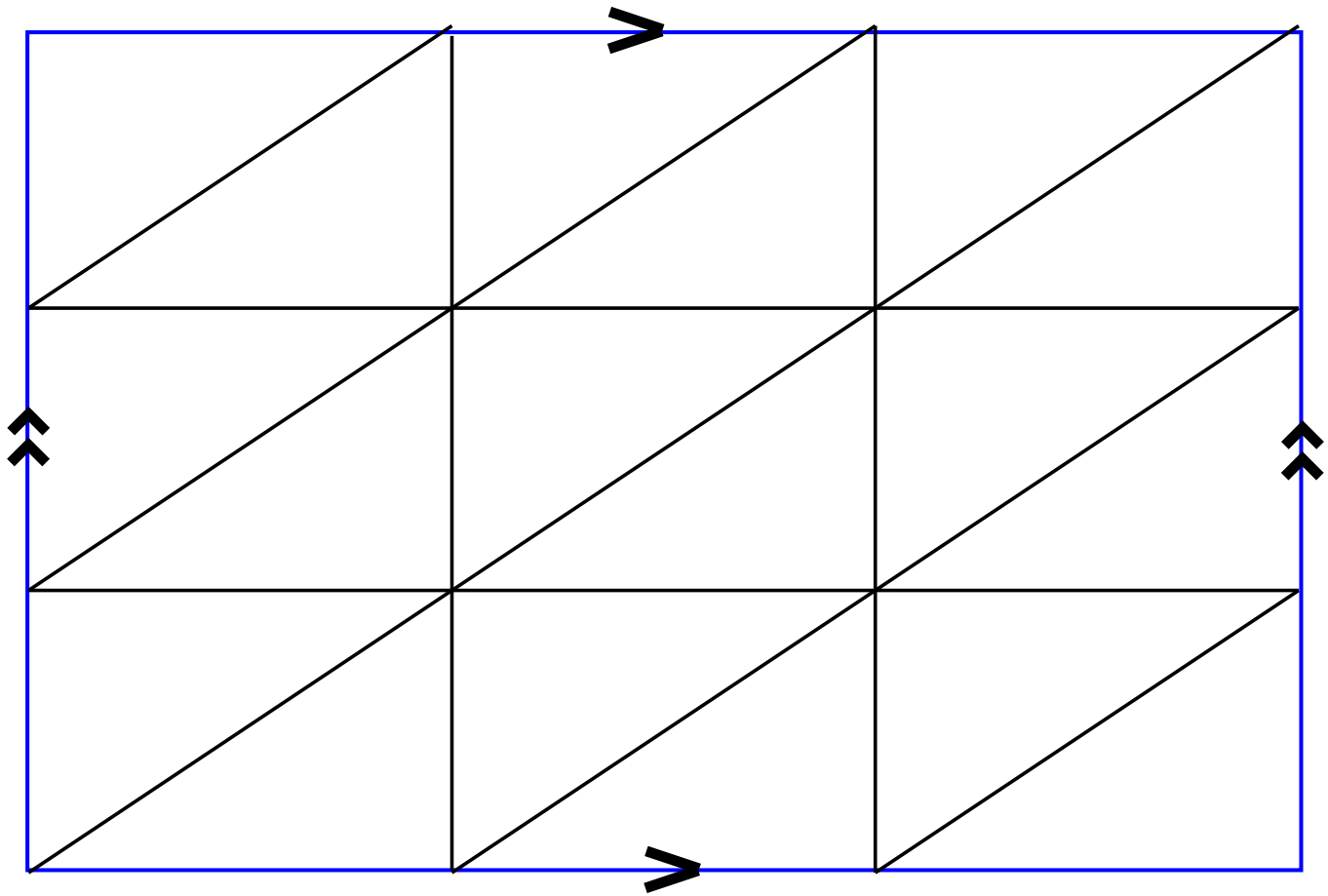
The Torus



Constructing Torus

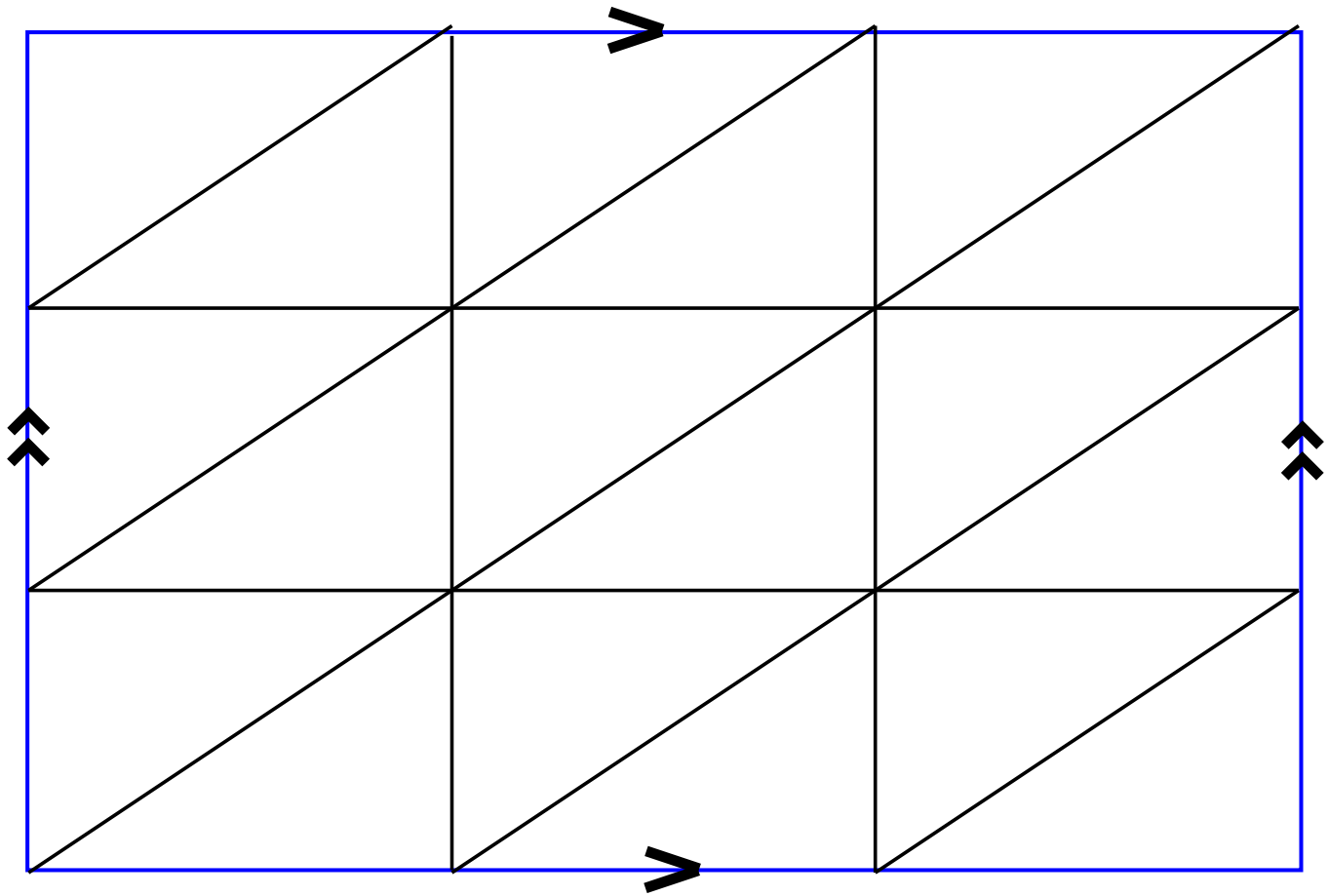
Show Video

Torus Fundamental Polygon



Torus Fundamental Polygon

- What is the Euler characteristic of a torus?



Intermezzo: Rhythm And Tori / Grateful Dead

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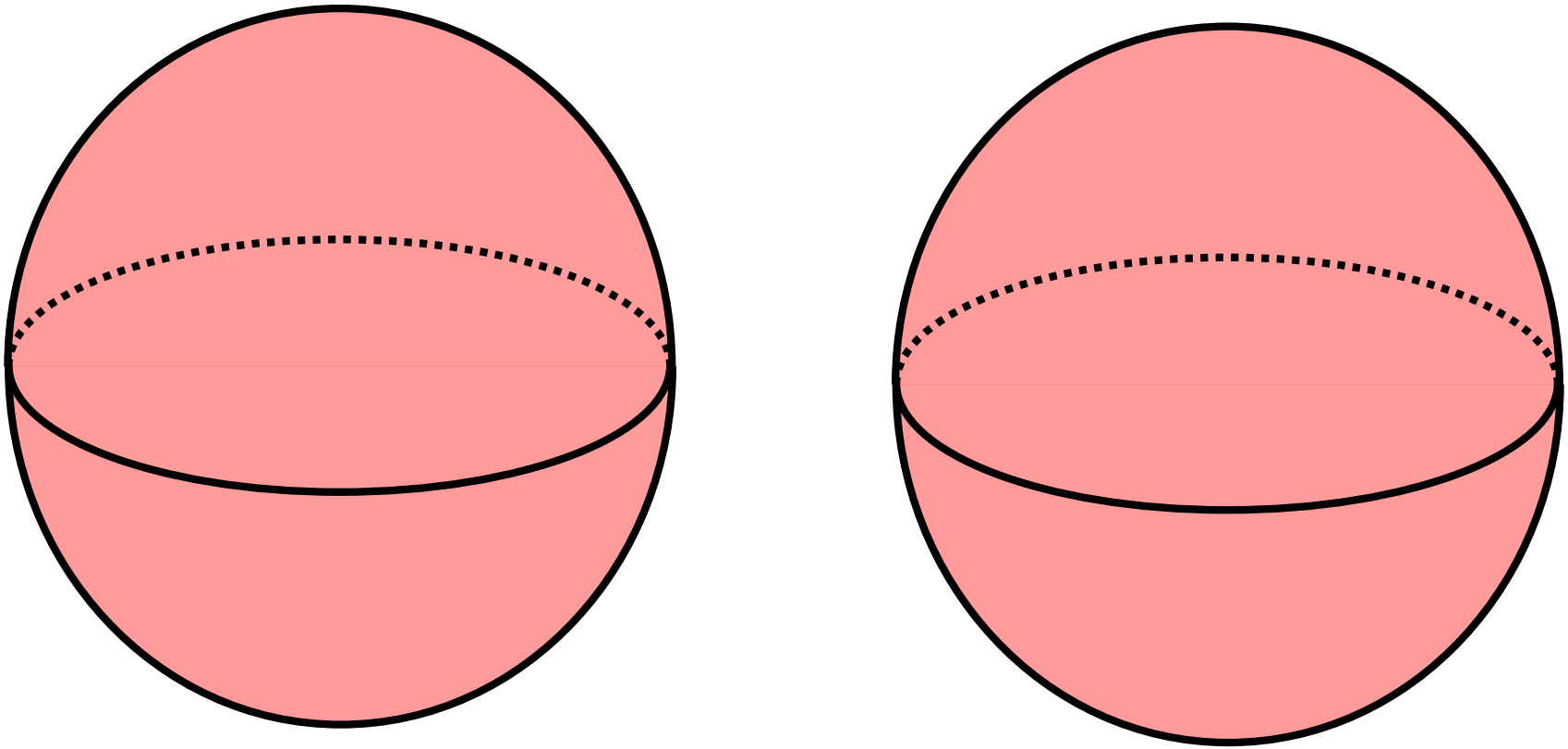
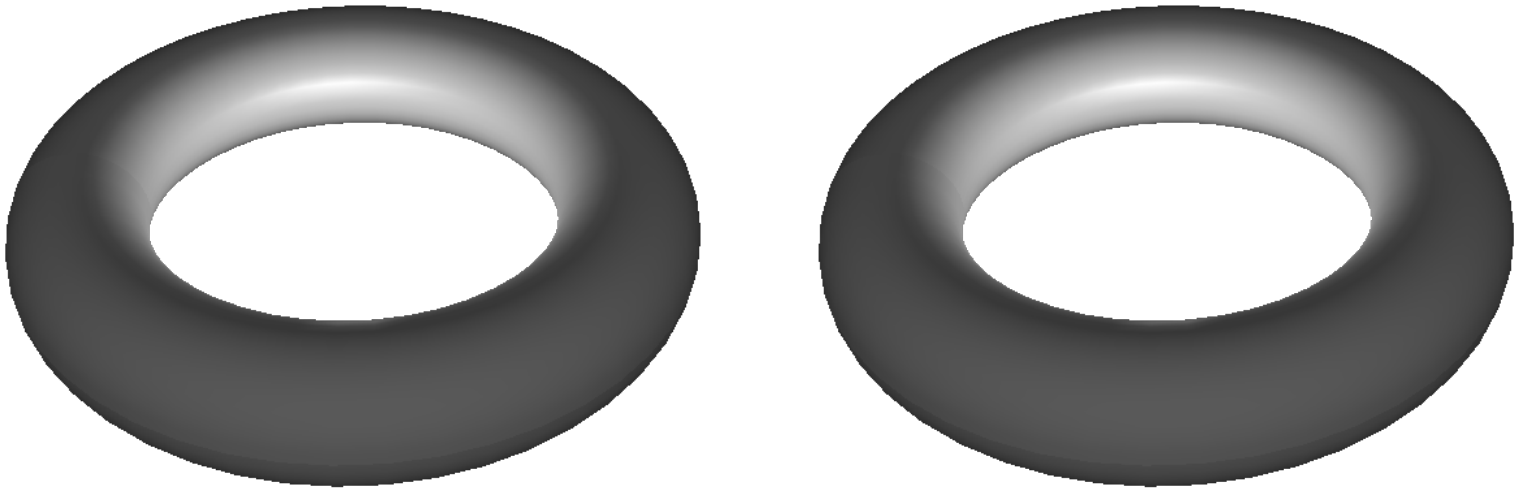
Duplicating Spheres

What's the euler characteristic of two spheres?



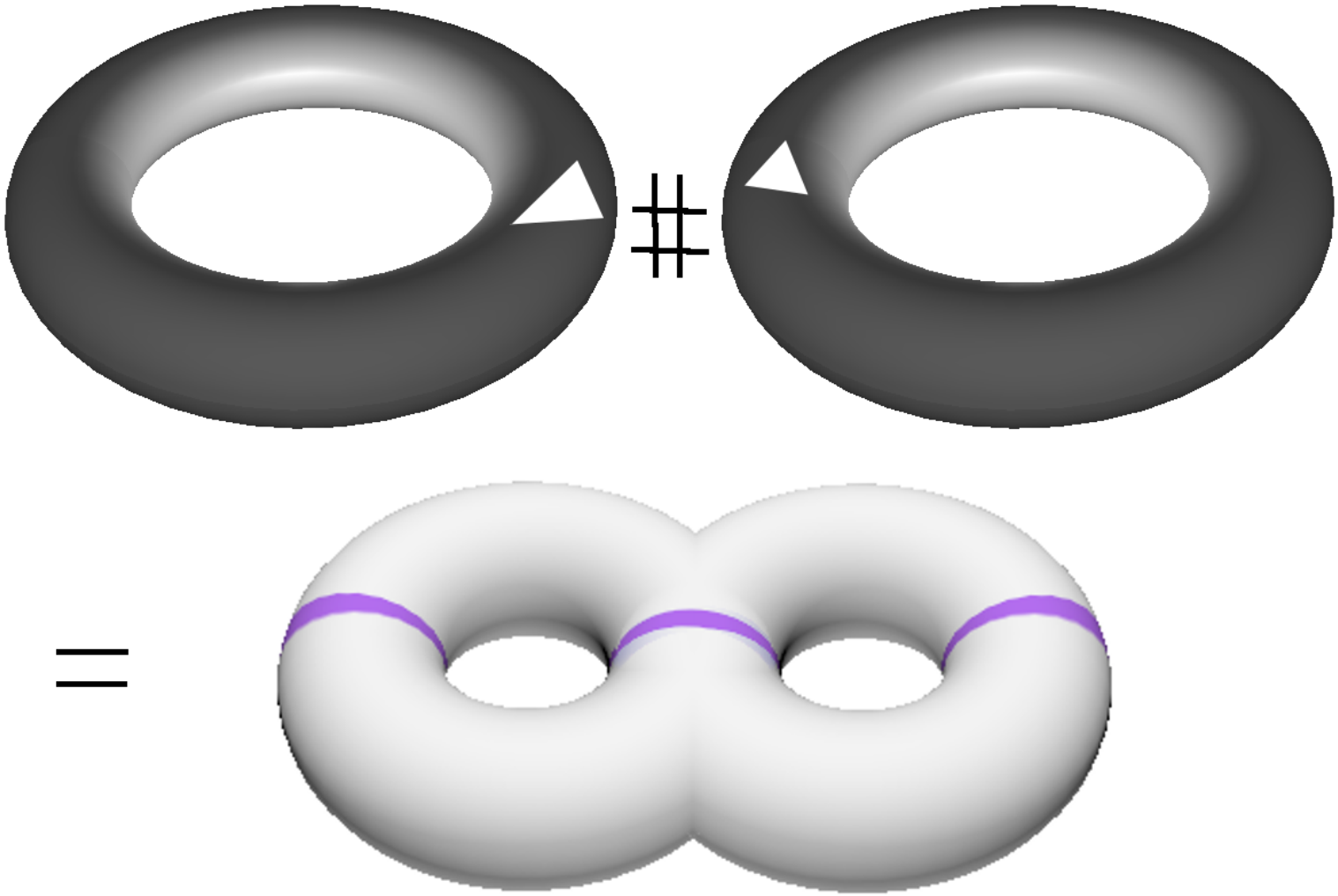
Duplicating Tori

What's the euler characteristic of two tori?



Connected Sum

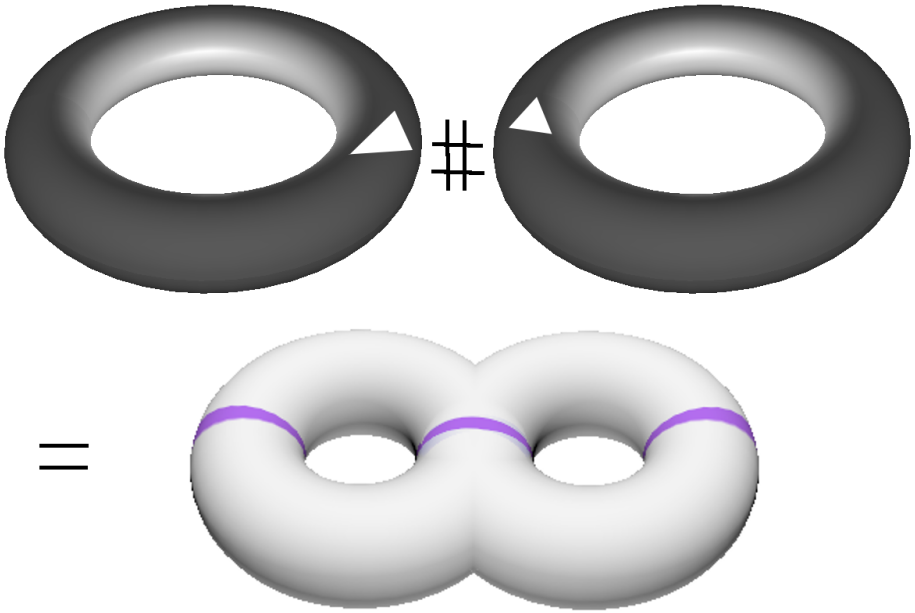
$T_1 \# T_1 = T_2$



Connected Sum

$T_1 \# T_1 = T_2$

What is the Euler characteristic?



Connected Sum: g Tori

What is the Euler characteristic of $T_N = T_1 \# T_1 \# \ldots \# T_1$ g times?

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$$\chi = 2 - 2g$$

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$$\chi = 2 - 2g$$

- g is known as the "genus"

Connected Sum with Spheres

What is the connected sum of a sphere with a sphere?

Connected Sum with Spheres

What is the connected sum of a torus with a sphere?

Euler Characteristic: Homology

$$\chi = \beta_0 - \beta_1 + \beta_2$$

- β_0: Number of connected components
- β_1: Number of independent loops/cycles
- β_2 Number of independent voids