Lecture 25: Geodesic Paths

COMPSCI/MATH 290-04

Chris Tralie, Duke University

4/14/2016

Announcements

- Group Assignment 3 Out: First Deadline Monday 4/18. Final Deadline Tuesday 4/26
- Final Project Final Deadline 5/3 5:00 PM

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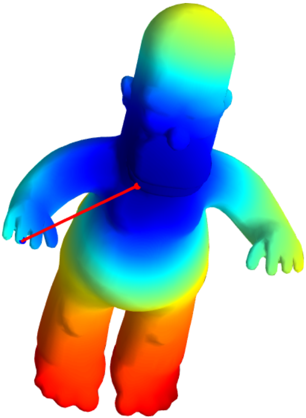
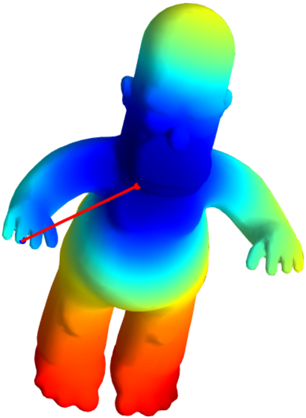
Geodesic Paths



Euclidean Path (shortest path of flying fly)

Geodesic Paths



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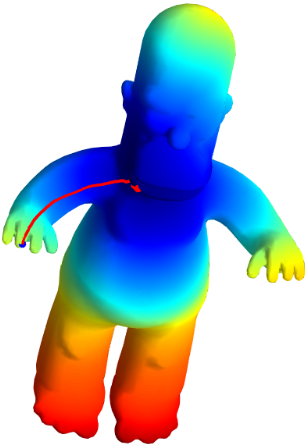
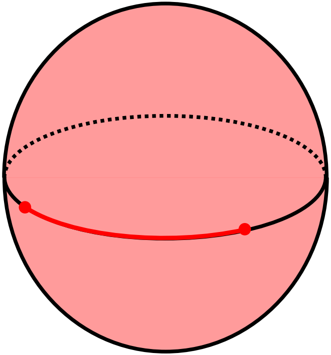
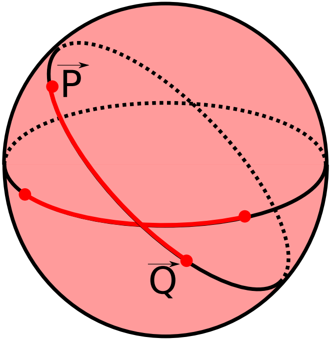


Geodesic Path (shortest path of crawling ant)

Geodesic Paths on Spheres

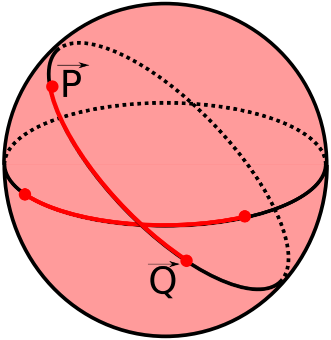


Geodesic Paths on Spheres



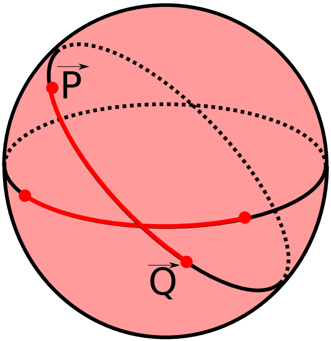
- ▷ Geodesic paths on spheres lie along great circles

Geodesic Paths on Spheres



- ▷ Geodesic paths on spheres lie along great circles
- ▷ Geodesic distance is the shortest geodesic path

Geodesic Paths on Spheres



- Geodesic paths on spheres lie along great circles
- Geodesic distance is the shortest geodesic path
- *What is the geodesic distance between two points $\vec{P}$ and $\vec{Q}$ on a sphere centered at the origin with radius R?*

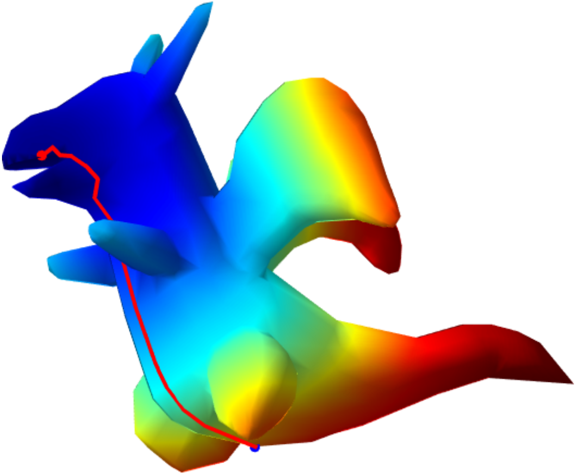
Geodesic Paths on Spheres

What is the geodesic distance between two points $\vec{P}$ and $\vec{Q}$ on a sphere centered at the origin with radius R?

$$R\cos^{-1}\left(\frac{\vec{P}\cdot\vec{Q}}{||\vec{P}||||\vec{Q}||}\right) = R\cos^{-1}\left(\frac{\vec{P}\cdot\vec{Q}}{R^2}\right)$$

Remember SLERP??

Another Geodesic Mesh Example



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Dijkstra’s Algorithm Review

```
def Dijsktra(Graph, source):
    list dists
    list prev
    dist[source] = 0
    Queue Q
    for vertex v in Graph:
        if v not source:
            dists[v] = Infinity
            prev[v] = Undefined
        Q.add(v, dists[v])
    while len(Q) > 0:
        u = Q.getMin()
        for v in neighbors(u):
            d = dists[u] + length(u, v)
            if d < dists[v]:
                dists[v] = d
                prev[v] = u
                Q.decreasePriority(v, d)
    return (dist, prev)
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Dijkstra's Algorithm Review

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What is the worst case behavior for

- V vertices
- E edges

for a balanced min heap Q?

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What is the worst case behavior for

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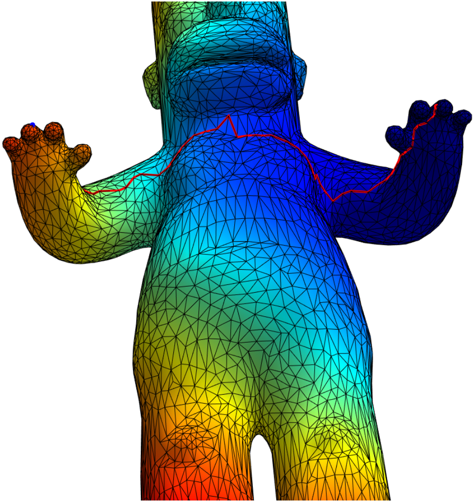
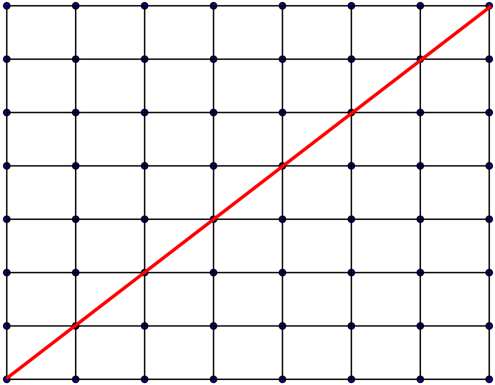
$O((E+V)\log(V))$

Dijkstra's Directly on Mesh Edges



Dijkstra's Directly on Mesh Edges

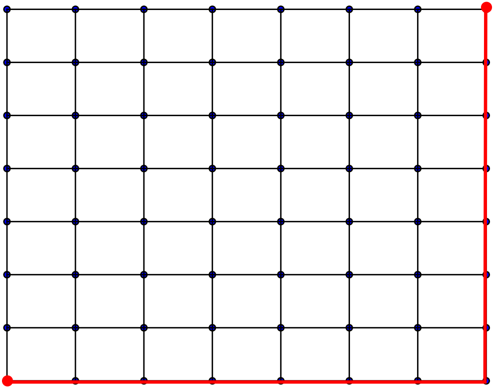
8x8 Cartesian Grid: Side Length 1



Shortest path along mesh is length $7\sqrt{2}$

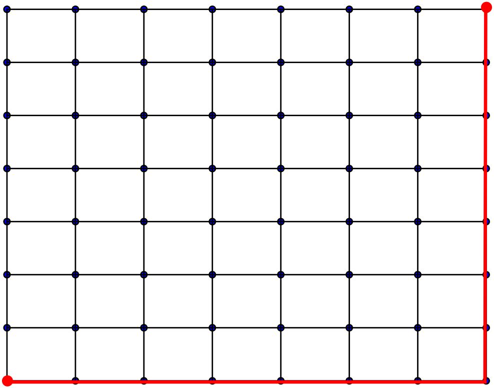
Dijkstra's Directly on Mesh Edges

8x8 Cartesian Grid: Side Length 1



Dijkstra's Directly on Mesh Edges

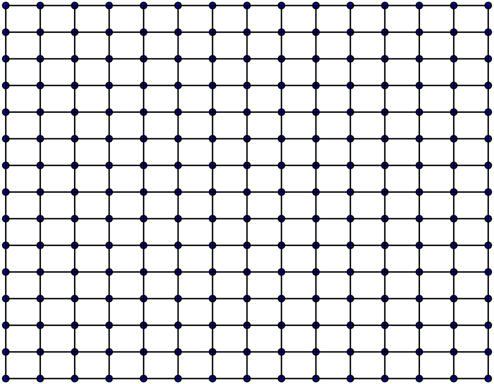
8x8 Cartesian Grid: Side Length 1



Shortest path along mesh is 14

Dijkstra's Directly on Mesh Edges

Does refining the grid help?
15x15 Cartesian Grid: Side Length 0.5



Dijkstra’s Directly on Mesh Edges

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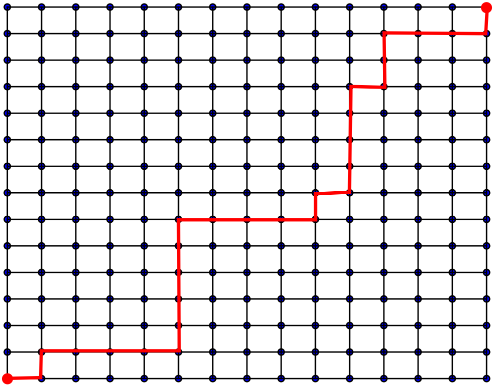
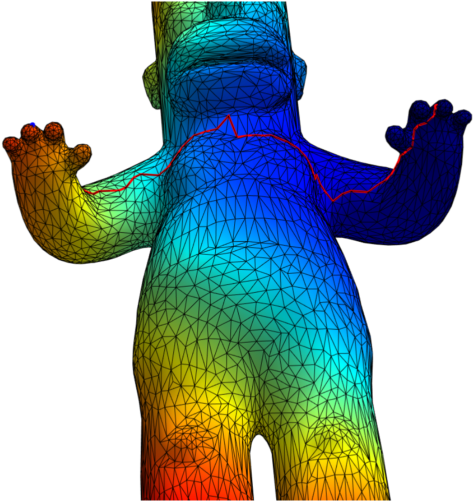


Nope!

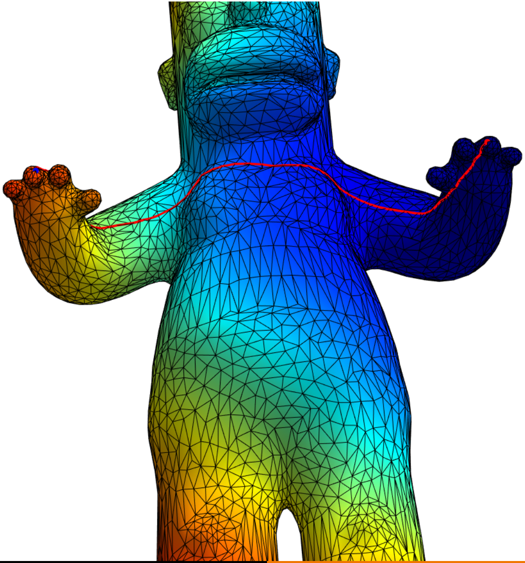
Dijkstra’s Directly on Mesh Edges

In general, mesh biases the solution!



Fast Marching

A modification of Dijkstra's algorithm to cut through triangles



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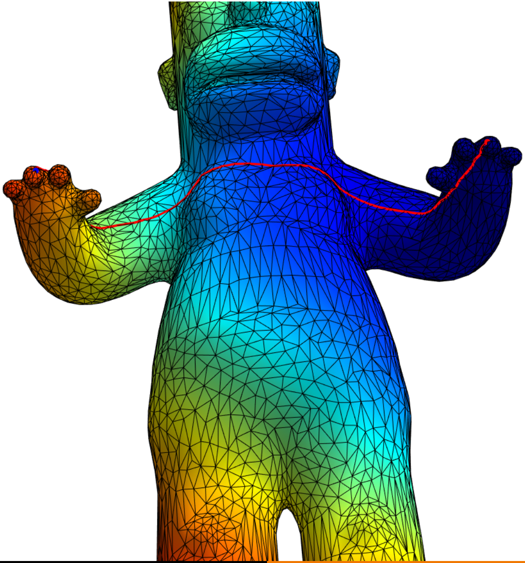

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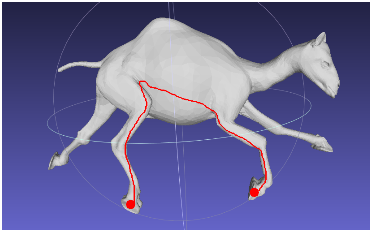
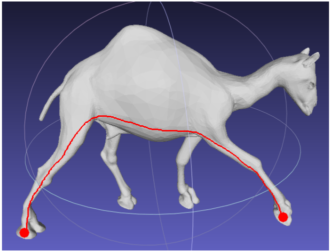
Mesh Isomorphisms

An isomorphism preserves all pairwise geodesic distances





Mesh Isomorphisms

Contrast with Euclidean