Lecture 26: MDS / Canonical Forms

COMPSCI/MATH 290-04

Chris Tralie, Duke University

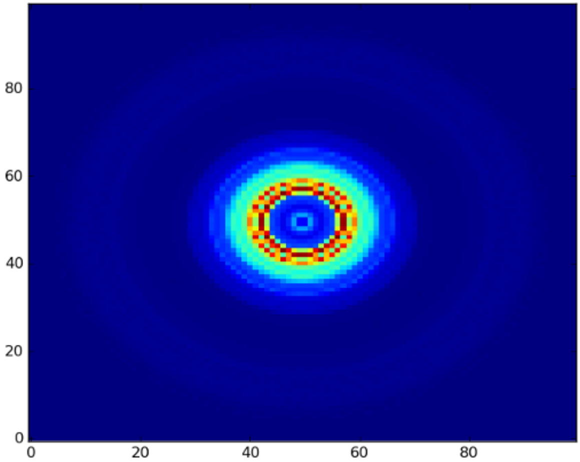
4/19/2016

Announcements

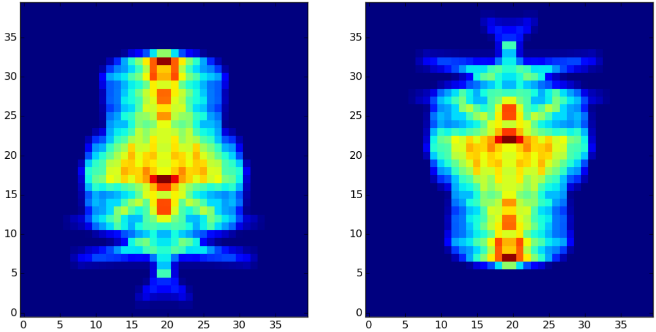
- Group Assignment 3 Final Deadline Tuesday 4/26
- Guest Lecture Thursday
- No office hours Thursday

Spin Images

Why did they all look so boring and unlike the objects in question?



Spin Images



I made a mistake on the assignment! First principal axis is vertical axis in image

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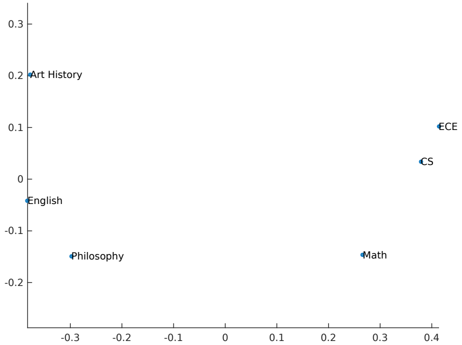
Academic Majors Distances: Your Choices

	Art History	English	Math	CS	ECE	Philosophy
Art History	0	0.36	0.74	0.8	0.81	0.37
English	0.36	0	0.7	0.8	0.82	0.29
Math	0.74	0.7	0	0.31	0.32	0.59
CS	0.8	0.8	0.31	0	0.2	0.71
ECE	0.81	0.82	0.32	0.2	0	0.77
Philosophy	0.37	0.29	0.59	0.71	0.77	0

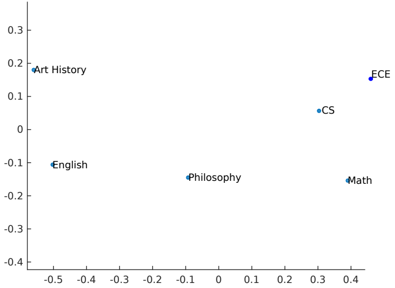
Multidimensional Scaling down to $\mathbb{R}^2$



Academic Majors Distances: Chris's Choices

	Art History	English	Math	CS	ECE	Philosophy
Art History	0	0.3	1	0.9	1	0.5
English	0.3	0	0.9	0.8	1	0.3
Math	1	0.9	0	0.3	0.2	0.3
CS	0.9	0.8	0.3	0	0.1	0.4
ECE	1	1	0.2	0.1	0	0.6
Philosophy	0.5	0.3	0.3	0.4	0.6	0

Multidimensional Scaling down to $\mathbb{R}^2$



Multidimensional Scaling

- Given an $N \times N$ symmetric discrete similarity matrix D (i.e. $D_{ij} = D_{ji}$)
- Given a Euclidean dimension K

Find a point cloud $X \in \mathbb{R}^{N \times K}$ so that

$$D_{ij} \approx \sqrt{\sum_{k=1}^{K} (X[i,k] - X[j,k])^2}$$

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In other words, find a point cloud in Euclidean K-space that *best approximates* the distances

MDS: Euclidean Dimension Reduction

$$D_{ij} \approx \sqrt{\sum_{k=1}^{K} (X[i,k] - X[j,k])^2}$$

What if D_{ij} comes from a Euclidena space of dimension $d > k$? Can we solve this using something else we learned in the course?

MDS: Euclidean Dimension Reduction

$$D_{ij} \approx \sqrt{\sum_{k=1}^{K}(X[i,k] - X[j,k])^2}$$

What if D_{ij} comes from a Euclidena space of dimension $d > k$?
Can we solve this using something else we learned in the course?
This is equivalent to PCA!! If we let $k = d$, then we can represent distances exactly

MDS: Non-Euclidean Space Reduction

Can we always find a point cloud that satisfies a given D by making k arbitrarily high?

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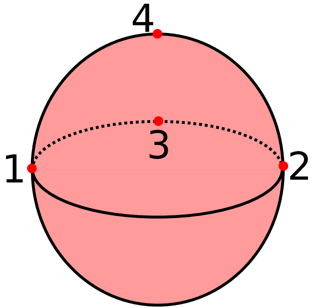
Assume sphere of radius $2/\pi$ with points in the following configuration:



	v_1	v_2	v_3	v_4
v_1				
v_2				
v_3				
v_4				

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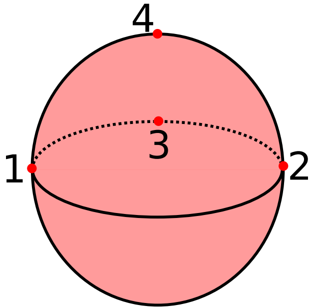

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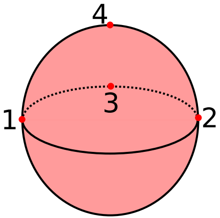

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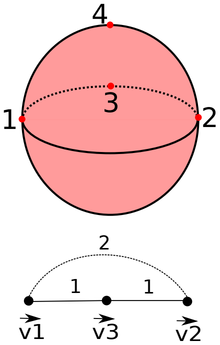

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v_1, v_3, v_2 along a line

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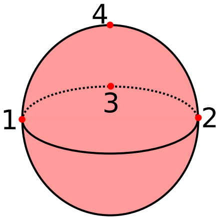

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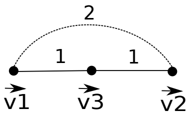
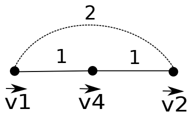
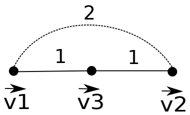


v_1, v_3, v_2 along a line

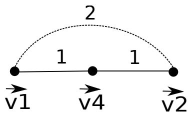


v_1, v_4, v_2 also along line!

MDS: Non-Euclidean Space Reduction



v_1, v_3, v_2 along a line



v_1, v_4, v_2 also along line!

This implies that v_4 and v_3 must collapse to the same point in any Euclidean space.

- In other words, distances along the sphere cannot be perfectly realized using a Euclidean space of any finite dimension!

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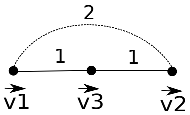

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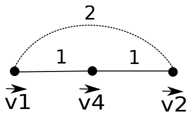

v_1, v_4, v_2 also along line!

This implies that v_4 and v_3 must collapse to the same point in any Euclidean space.

- In other words, distances along the sphere cannot be perfectly realized using a Euclidean space of any finite dimension!
- (But let's do our best and see what we come up with)

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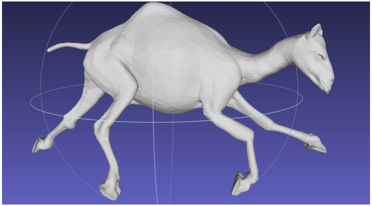
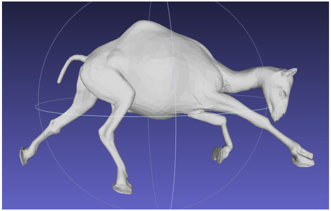
Nonrigid Shape Alignment

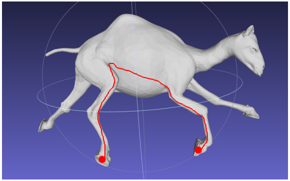
How do I align these two camels??

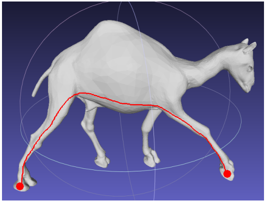




Geodesic Distances

Geodesic distances are invariant to *isometries* (aka bending without stretching)





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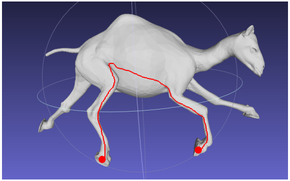
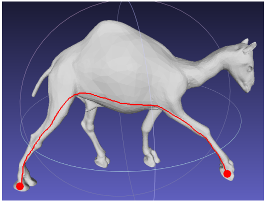
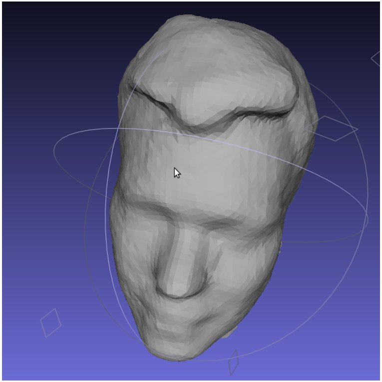




What if we try to apply MDS to the distance matrix we get from geodesic distances?

Face Example



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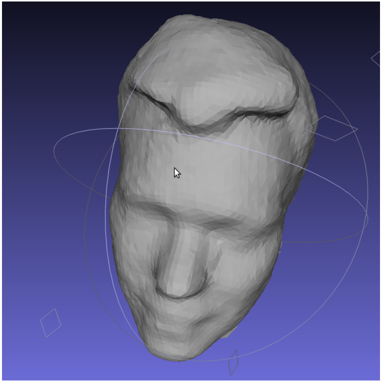

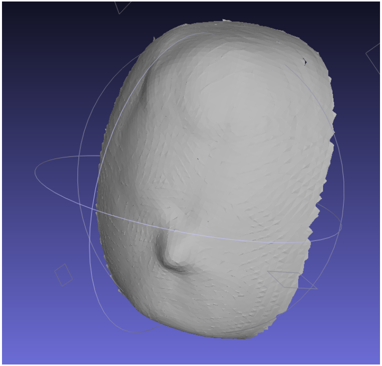

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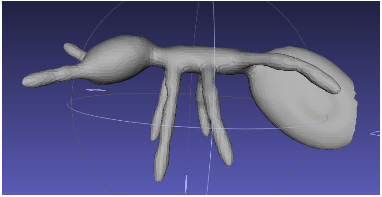

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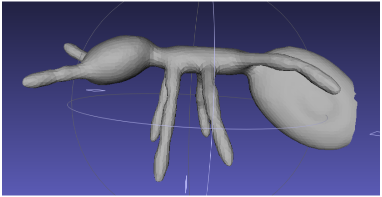
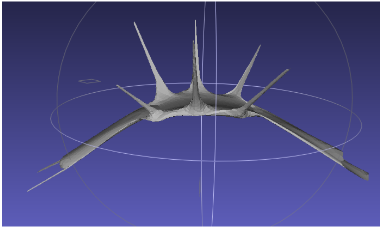
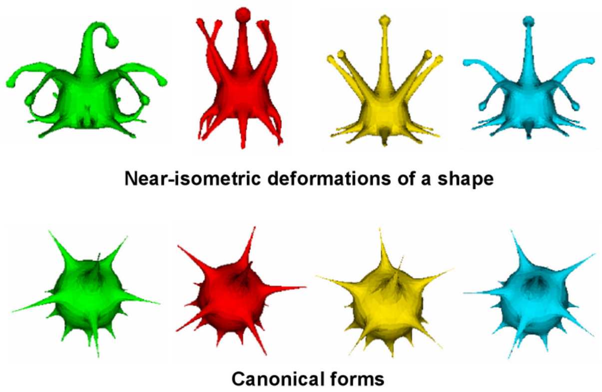


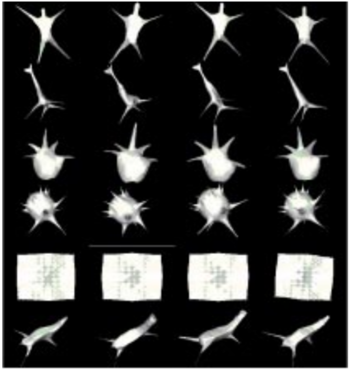


More Examples



Bronstein

Lots More Examples



Elad Kimmel 2001: “Bending Invariant Representations for Surfaces”