Lecture 4: Planes, Interior Point Testing, Duality

COMPSCI/MATH 290-04

Chris Tralie, Duke University

1/26/2016

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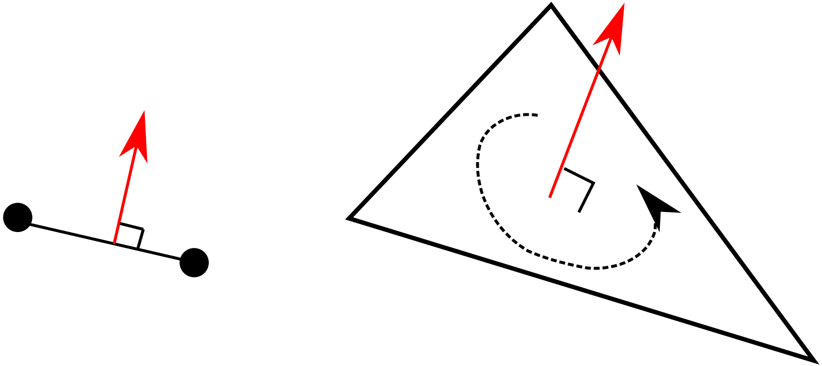
- Duality

Announcements

- Everyone got Mini 1 Part 1 in on time!
- Part 2 Due Friday 11:55 PM
- Drop/Add Tomorrow!
- SIGGRAPH Student Volunteers application
  `http://s2016.siggraph.org/student-volunteers`

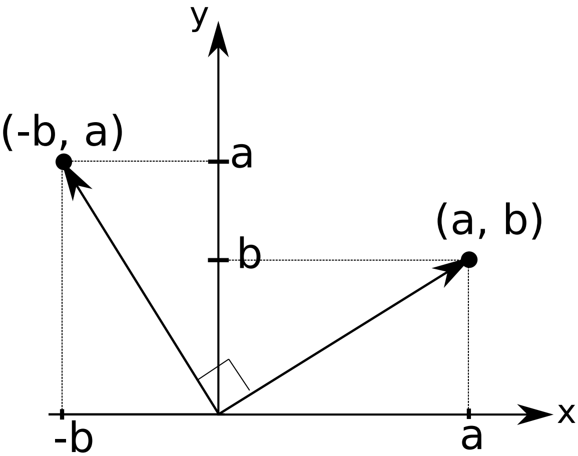
Normals

- Vector in direction perpendicular to object in question



Perpendicular To A 2D Vector

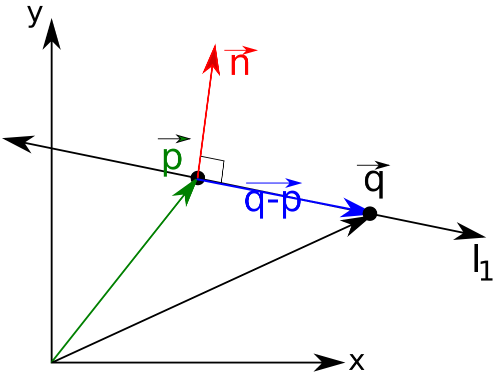
- Negate y and swap: $(a, b) \rightarrow (-b, a)$



Normal Form of A Line

Given point $\vec{p}$ and normal $\vec{n}$, a point $\vec{q}$ is on line if

$$(\vec{q} - \vec{p}) \cdot \vec{n} = 0$$

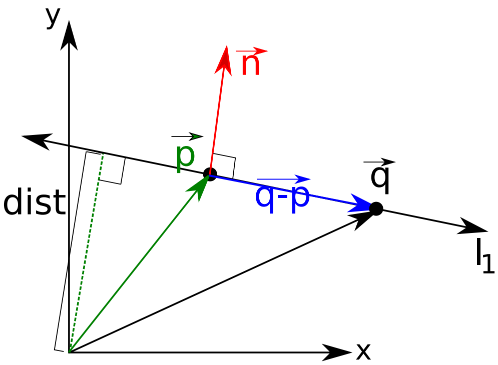


Normal Form of A Line

$$(\vec{q} - \vec{p}) \cdot \vec{n} = 0$$

Assume $||\vec{n}|| = 1$ (unit normal)

$$\vec{q} \cdot \vec{n} = \vec{p} \cdot \vec{n} = d(l_1, \text{origin})$$

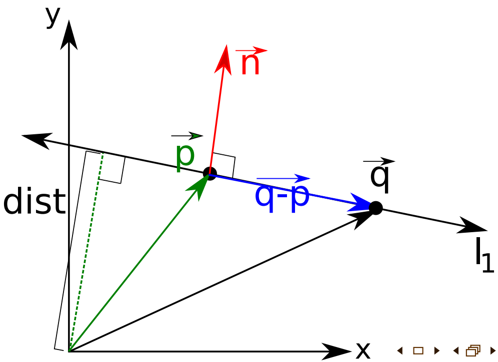


Normal Form of A Line

$$(\vec{q} - \vec{p}) \cdot \vec{n} = 0$$

Let $q = (x, y)$. Expanding and rewriting as implicit linear equation

$$n_x x + n_y y - d(l_1, \text{origin}) = 0$$



Line: Degrees of Freedom

$$n_x x + n_y y - d(l_1, \text{origin}) = 0$$

$$Ax + By + C = 0$$

Implicit form

- ▷ How many *degrees of freedom* are there in a line?

Normal Form of a Line

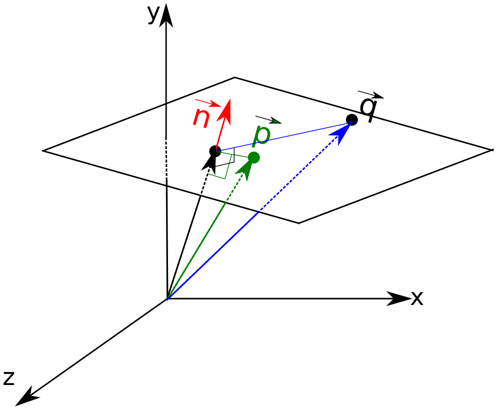
- What's the line normal of the line $y = mx + b$?

2D Planes

Given point $\vec{p}$ and normal $\vec{n}$, a point $\vec{q}$ is on plane if

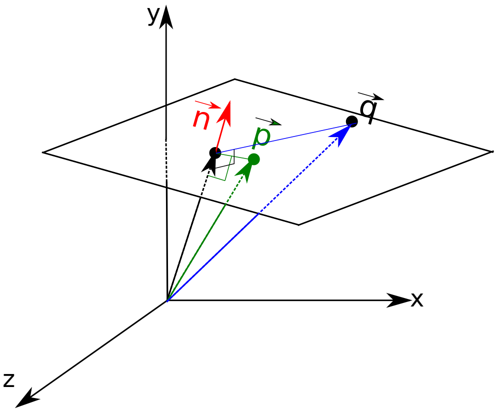
$$(\vec{q} - \vec{p}) \cdot \vec{n} = 0 \implies \vec{q} \cdot \vec{n} = \vec{p} \cdot \vec{n} = 0$$

(now 3D vectors)



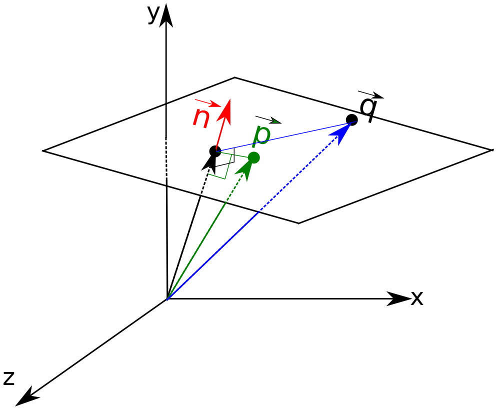
2D Planes

$$n_x x + n_y y + n_z z - d(p, \text{origin}) = 0$$

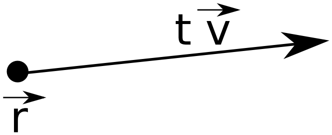


2D Planes

$$Ax + By + Cz + D = 0$$



Ray Intersect Plane



Plane: $(\vec{q} - \vec{p}) \cdot \vec{n} = 0$
Ray: $\vec{r} + t\vec{v}, t \geq 0$

$$t = \frac{-\vec{r} \cdot \vec{n}}{\vec{v} \cdot \vec{n}}$$

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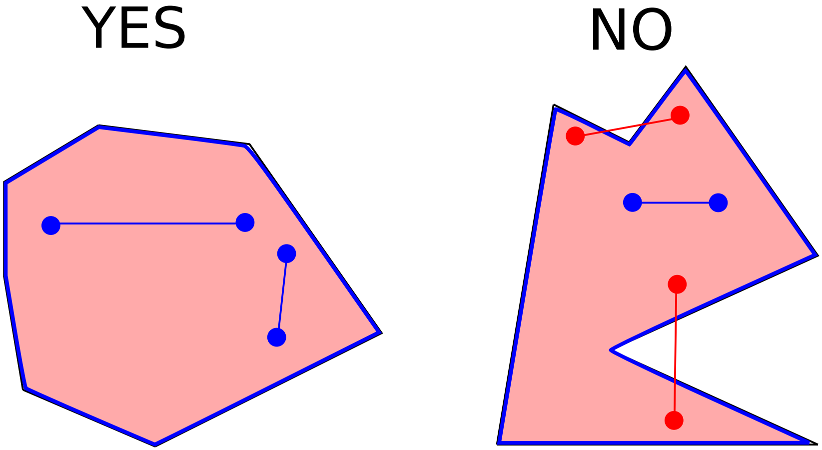
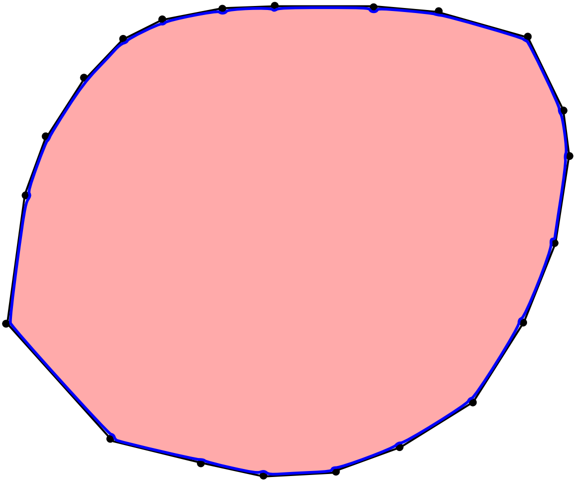
Convex Polygons

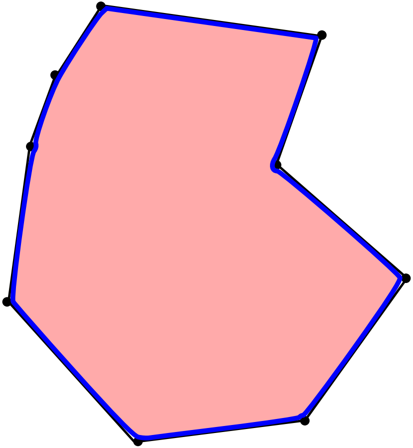


Definition extends to 3D polytopes (and any geometric set)

Convex Or Not?



Convex Or Not?



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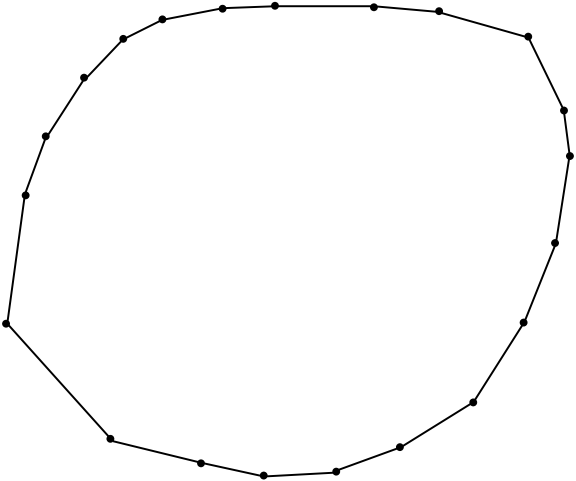
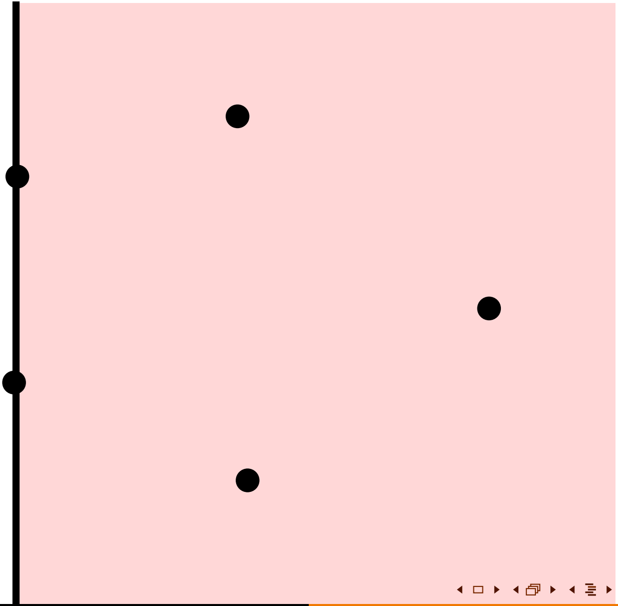


Point Inside Convex Polygon: Halfplane Method



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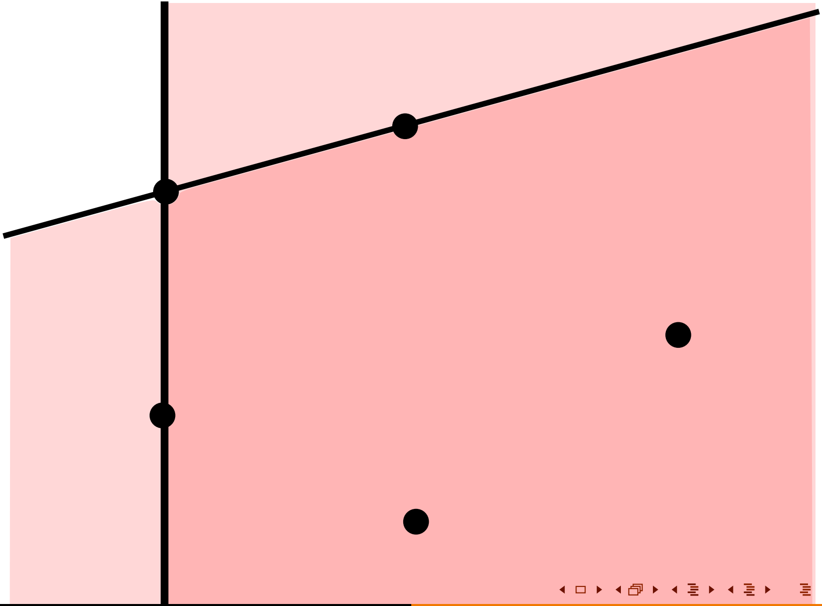

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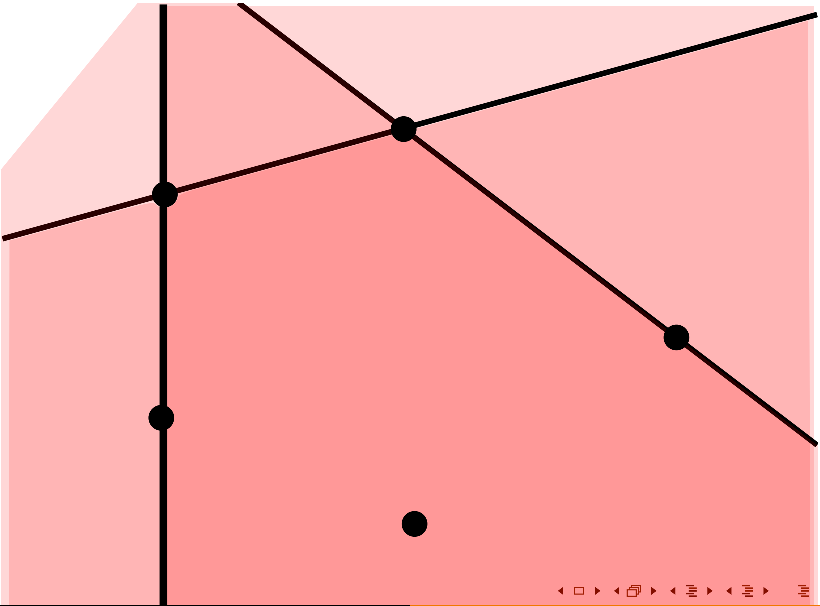

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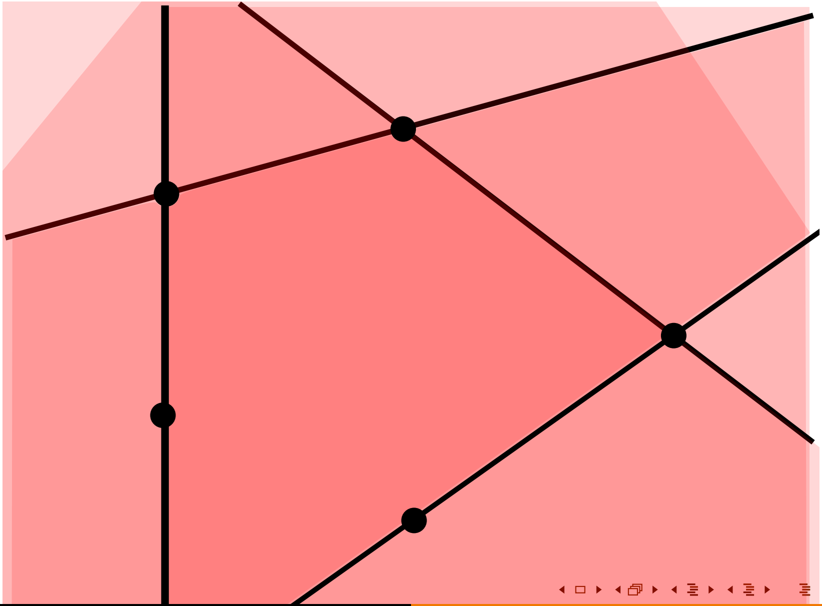

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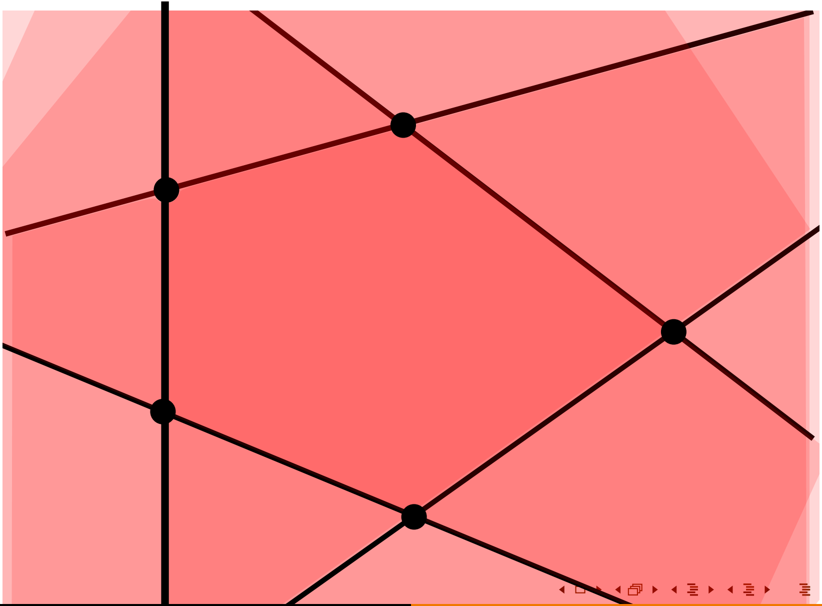
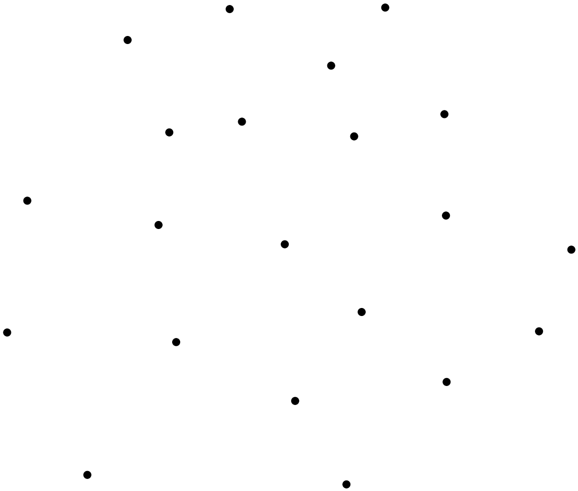
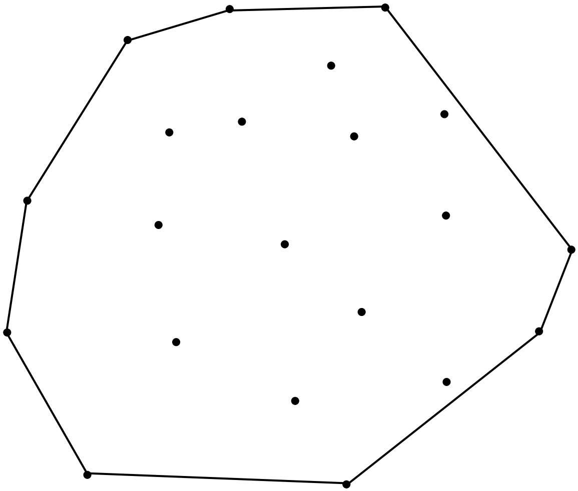


Convex Hull (Segue)

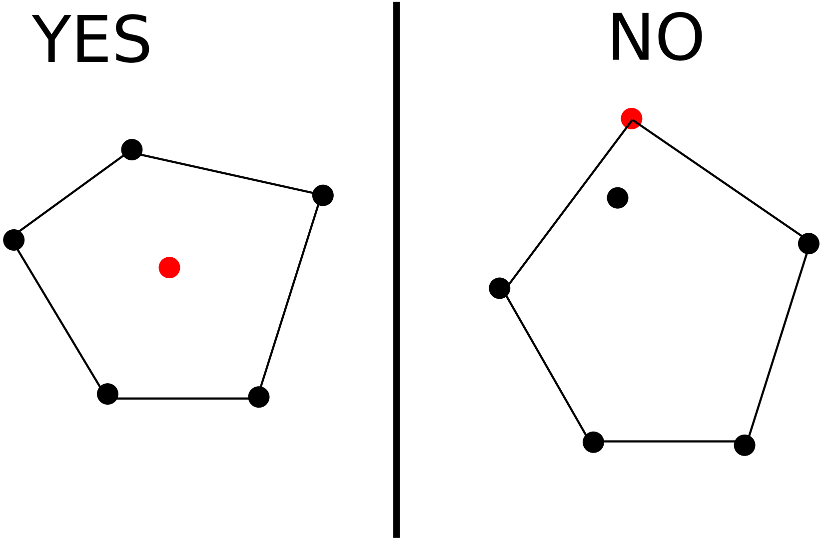


Point Inside Convex Polygon: Hull Method

Convex Hull (Segue)

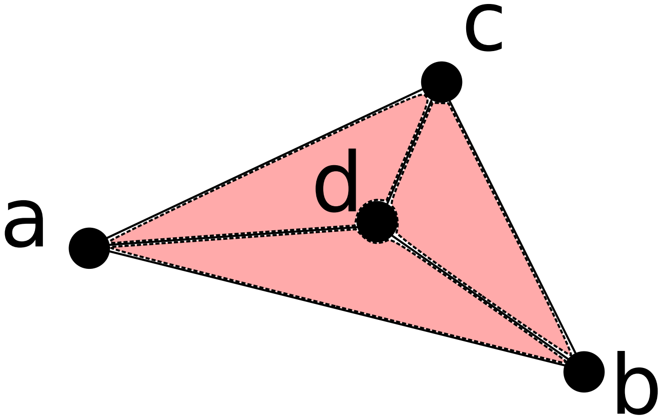


Convex Hull Test



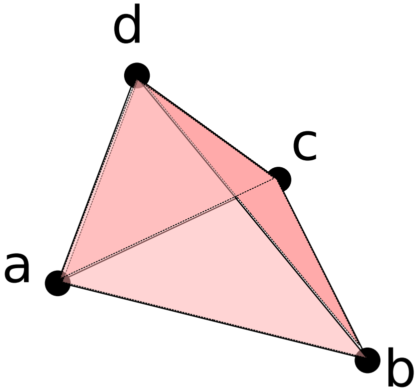
Point Inside Convex Polygon: Area Method

$$\text{Area}(\triangle abc) = \text{Area}(\triangle abd) + \text{Area}(\triangle bcd) + \text{Area}(\triangle cad)$$



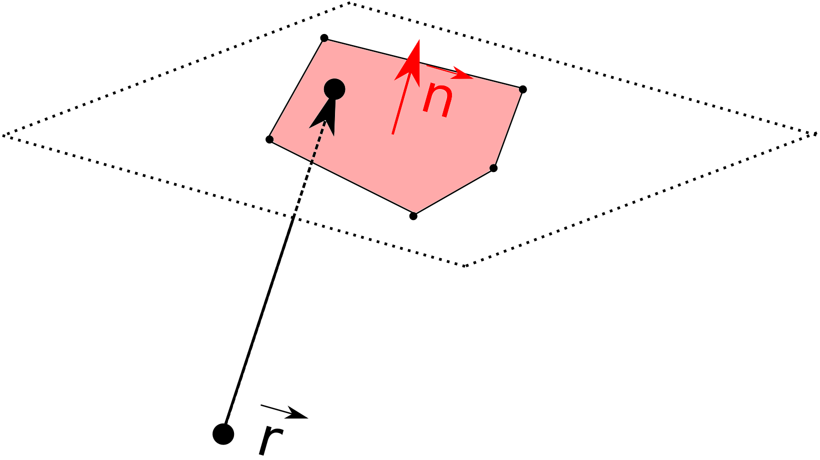
Point Inside Convex Polygon: Area Method

$$\text{Area}(\triangle abc) < \text{Area}(\triangle abd) + \text{Area}(\triangle bcd) + \text{Area}(\triangle acd)$$

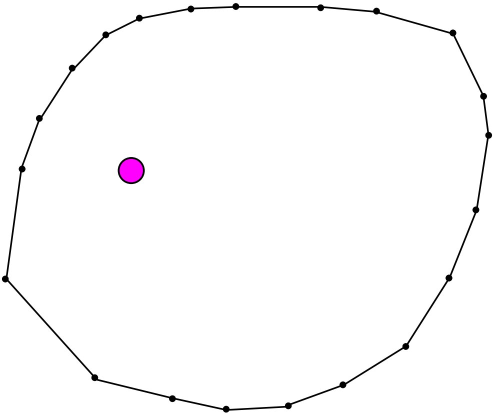


Point Inside Convex Polygon: Area Method

3D Ray Convex Polygon Intersection



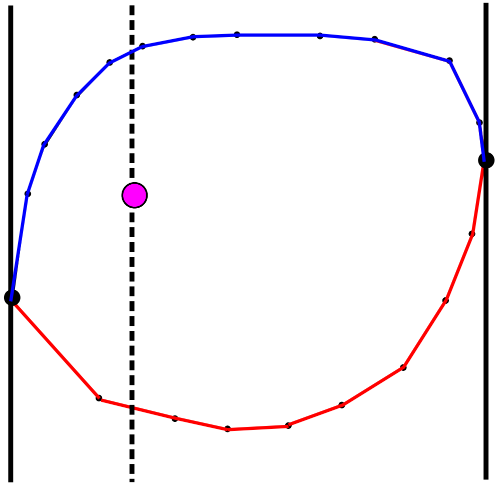
Logarithmic Convex Polygon Test



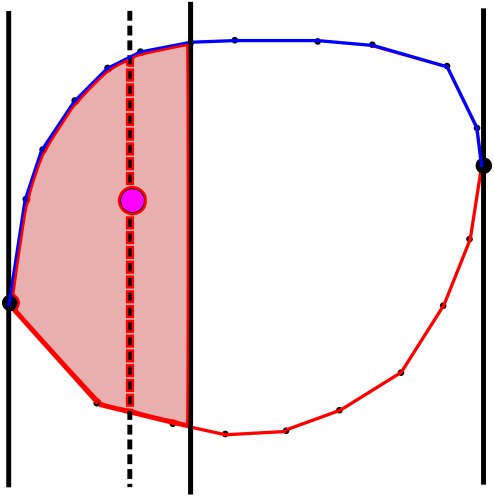
Logarithmic Convex Polygon Test

Segue: Binary Search

Logarithmic Convex Polygon Test



Logarithmic Convex Polygon Test



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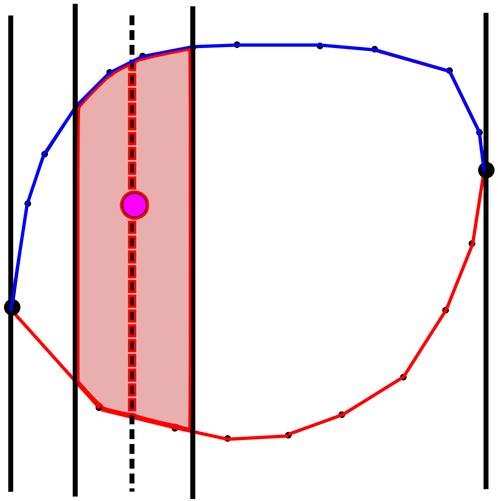

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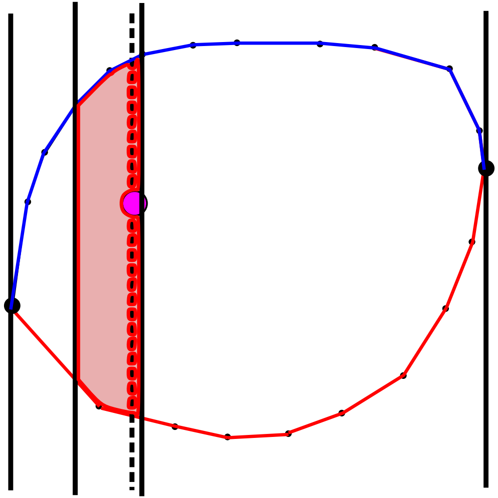

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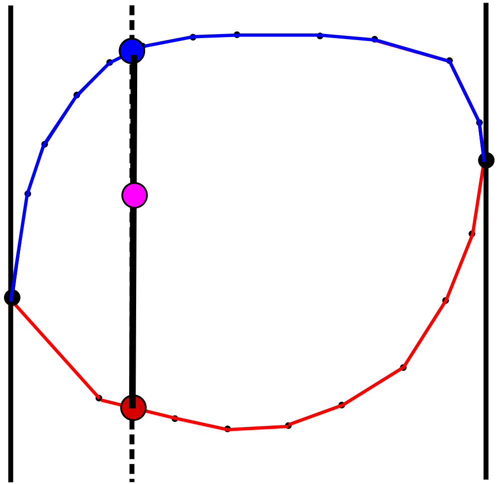
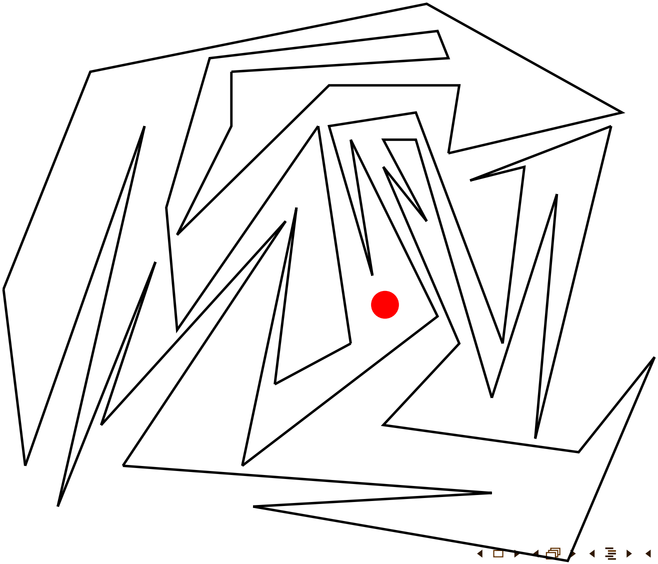
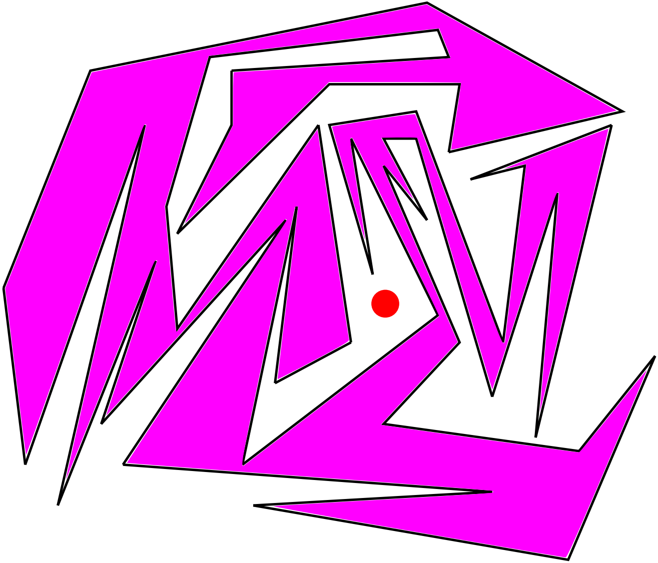


Nonconvex Polygons

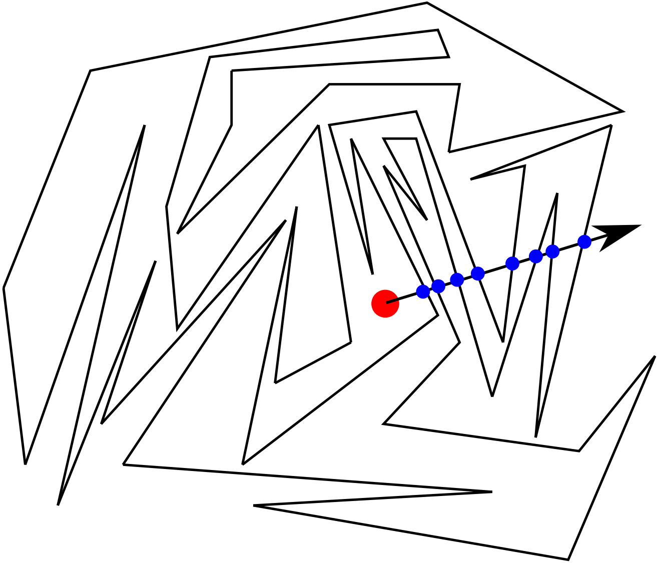
Inside or outside??



Nonconvex Polygons



Nonconvex Polygons: Ray Casting



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Points To Lines

$\vec{p}$: (a,b) $\longrightarrow$ p*: y = ax - b

l: y = cx + d $\longrightarrow$ $\vec{l}$*: (c,-d)

Points To Lines

$$\vec{p} > l \iff \vec{l^*} > p^*$$

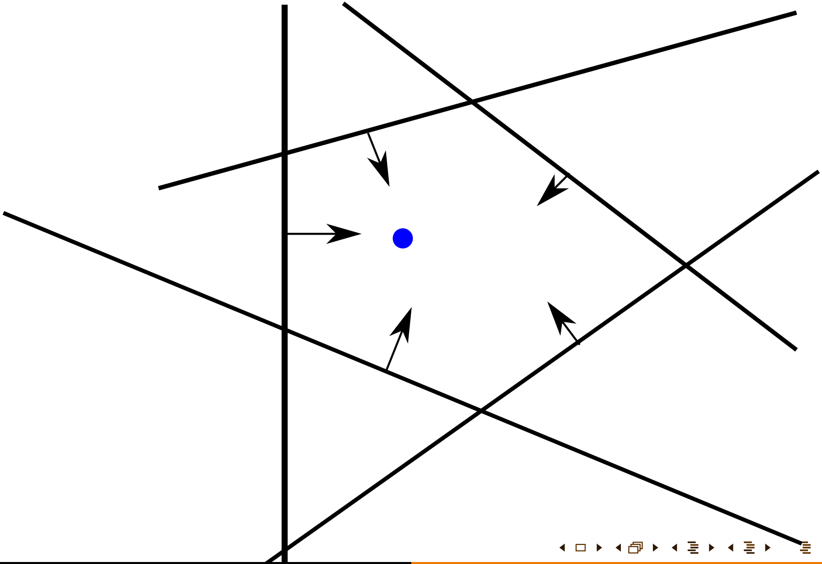
where " $>$ " means "above"
TODO: Verify this using vectors!

$\vec{p}$: (a,b) $\longrightarrow$ p*: y = ax - b

l: y = cx + d $\longrightarrow$ $\vec{l}$*: (c,-d)

Point Inside Convex Polygon: Halfplane Method

What dual problem did we solve??



Point Inside Convex Polygon: Halfplane Method

What dual problem did we solve??